A Short Note on Frequency-Domain Damage Calculation for Power Semiconductors Reliability Control Design

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Abstract

In this short note, we give a simple derivation of the damage calculation for power semiconductors based on the frequency-domain approximation of damage. This analysis can be further used to design linear control systems aimed at reducing the damage to the semiconductor switches in power converters.

Keywords: semiconductors lifetime, frequency-domain fatigue analysis

General calculation of the total damage of a stress signal using the Rainflow counting method is defined as follows (ASTM et al. (2011)):

$$\text{Damage} = \sum_i \frac{n(s_i)}{N(s_i)}, \tag{1}$$

where $n(s_i)$ is the number of cycles with the stress magnitude s_i, and $N(s_i)$ is the expected number of cycles to fail with the stress level s_i based on a pre-defined damage formula.

Now, if we assume that there are L cycles counted over the time frame of the stressor, then we can define the following expression as the damage intensity per time unit of a stress process:

$$D = \frac{L}{T} \sum_i \frac{p(s_i)}{N(s_i)}, \tag{2}$$

where T denotes the process duration, and $p(s_i)$ is the probability density function of the stress magnitude.

Replacing the sum by an integral and defining n_t as the number of cycles per unit time results in the following damage density calculation based on the Rainflow method:

$$D_{\text{RFC}} = n_t \int_0^\infty \frac{p(s)}{N(s)} ds. \tag{3}$$

If the stress signal is assumed to be Gaussian, and its power spectrum is considered narrow-band with the dominant central frequency of ω, then the damage term becomes

$$D_{\text{NB}} \approx \frac{\omega}{2\pi} \int_0^\infty \frac{\frac{s}{\sigma_s^2} e^{-\frac{s^2}{2\sigma_s^2}}}{N(s)} ds. \tag{4}$$

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Now, for the power semiconductor modules, we will use the lifetime model given in Arendt Wintrich (2021), except that for computational simplicity we neglect the low temperature swing extension in the model. Therefore, the damage density for the case of power modules becomes

$$D_{\mathrm{NB}} = \frac{\omega}{2\pi} \int_0^\infty \frac{\frac{x}{\sigma_s^2} e^{-\frac{s^2}{2\sigma_s^2}}}{A s^{-\alpha} e^{\frac{E_a}{k_b \bar{s}}} \left(C + \left(\frac{\omega}{2\pi}\right)^\gamma\right)} ds. \tag{5}$$

In the equation above, the stress signal is the junction temperature fluctuation, i.e. $s = \Delta T_j$, and $\bar{s}$ denotes the median of the junction temperature, T_j.

Removing the constant terms from the integral yields

$$\begin{aligned} D_{\mathrm{NB}} &= \frac{1}{2\pi A e^{\frac{E_a}{k_b \bar{s}}}} \frac{\omega}{\left(C + \left(\frac{\omega}{2\pi}\right)^\gamma\right)} \int_0^\infty s^\alpha \frac{s}{\sigma_s^2} e^{-\frac{s^2}{2\sigma_s^2}} ds, & (6) \\ &= \frac{1}{2\pi A e^{\frac{E_a}{k_b \bar{s}}}} \frac{\omega}{\left(C + \left(\frac{\omega}{2\pi}\right)^\gamma\right)} \left(\sqrt{2\sigma_s^2}\right)^\alpha \Gamma(1 + \frac{\alpha}{2}), & (7) \\ &= \frac{\Gamma(1 + \frac{\alpha}{2})}{2\pi A e^{\frac{E_a}{k_b \bar{x}}}} \frac{\omega}{\left(C + \left(\frac{\omega}{2\pi}\right)^\gamma\right)} \left(\sqrt{2 S_T(\omega) \Delta\omega}\right)^\alpha & (8) \end{aligned}$$

where Γ denotes the Gamma function. Note that in the last line, the variance of the temperature signal is replaced by its power spectral density (PSD), $S_T(\omega)$.

Using the “single moment” spectral approach method described in Benasciutti et al. (2013) to model the damage as a sum of damage contributions of narrow-band processes, the total damage is then computed by applying the following nonlinear summation rule:

$$D_{\mathrm{SM}} = \left(\sum_i D(\omega_i)^{\frac{2}{\alpha}}\right)^{\frac{\alpha}{2}} \propto \left(\sum_i \frac{\omega_i^{\frac{2}{\alpha}}}{\left(C + \left(\frac{\omega_i}{2\pi}\right)^\gamma\right)^{\frac{2}{\alpha}}} S_T(\omega_i) \Delta\omega\right)^{\frac{\alpha}{2}}. \tag{9}$$

Furthermore, the power spectral of the temperature fluctuation can be expressed in terms of the current profile through the following transfer functions:

$$S_T(\omega) = |G_{TP}(j\omega)|^2 \left|G_{PI_{ref}}(j\omega)\right|^2 S_{I_{ref}}(\omega), \tag{10}$$

where $S_{I_{ref}}(\omega)$ denotes the PSD of the reference input current profile, $G_{PI_{ref}}(j\omega)$ denotes the closed-loop transfer function from the reference current profile to the semiconductor power loss[1], $G_{TP}(j\omega)$ is the transfer function from the power loss to the resulting temperature profile[2].

Plugging (10) in (9) leads to

$$D_{\mathrm{SM}} \propto \left(\sum_i |W(\omega_i)|^2 \left|G_{PI_{ref}}(j\omega)\right|^2 S_{I_{ref}}(\omega) \Delta\omega\right)^{\frac{\alpha}{2}}, \tag{11}$$

where W is a frequency-dependent weight defined as

$$W(\omega) := \frac{\omega^{\frac{1}{\alpha}}}{\left(C + \left(\frac{\omega}{2\pi}\right)^\gamma\right)^{\frac{1}{\alpha}}} G_{TP}(j\omega). \tag{12}$$

The magnitude of W is plotted in Fig. 1. The lifetime model parameters are given in Arendt Wintrich (2021), and the thermo-electrical parameters of the transfer function G_{TP} are approximately estimated for a SiC MOSFET module given in Semikron Danfoss (2025).

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[1] Assuming a linear or linearized model. For IGBTs, the conduction loss is approximately linear in I.

[2] The thermo-electrical model is considered to be a linear RC system. See Mohan et al. (2003) for details.

As we can see, at low frequencies the damage accumulation is relatively lower because of less number of cycles at these frequencies. At high frequencies, the temperature fluctuation is limited due to the bandwidth of the thermodynamics and thus the damage becomes lower. Note that the controller dynamics are embedded in $G_{PI_{ref}}$, and therefore to reduce the damage to the semiconductor modules, the controller has to be optimally designed to fulfill the performance requirement, i.e. achieving the desired current profile, while avoiding high damage accumulation. The weight function can be approximated by linear systems to be used in the linear control system design procedures such as $H2/H_\infty$.

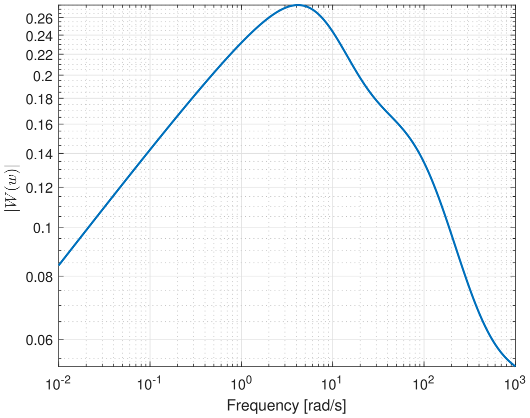

Figure 1: The $W(\omega)$ evaluated at different frequencies, reflecting the contribution of damage at each frequency.

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