2023-10-10

How Do Students Use Basic Aspects Of Functions When Learning Mathematics In A Chemistry Context?

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How do students use basic aspects of functional thinking when learning mathematics in a chemistry context?

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Conference Key Areas: *Innovative Teaching and Learning Methods, Fundamentals of Engineering: Mathematics and the Sciences*
Keywords: *math education, Life Sciences, functional thinking, digital learning environment*

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ABSTRACT

The mathematical concept of function is challenging for students in first-year undergraduate mathematics courses, especially when the concept is applied in the context of STEM courses. This difficulty is often due to a lack of conceptual understanding of functions. From a normative perspective, conceptual understanding of functions involves 1) dealing with the different representations of a function, namely table, graph, analytical term and verbal description, while 2) considering three different aspects of functions, namely correspondence, covariation and object. Previous research suggests that the covariation aspect is essential for achieving a sophisticated conceptual understanding of functions. In order to promote the conceptual understanding of functions, a digital self-learning environment was developed and implemented in the first-year basic mathematics course at the School of Life Sciences, University of Applied Sciences and Arts Northwestern Switzerland (FHNW). To facilitate the transfer of mathematical knowledge to applied STEM courses, the mathematical learning environment focuses on chemical reactions, where the concentration of the reactants is analysed. Initial findings from the qualitative content analysis show 1) how students use the different aspects of mathematical functions in the context of chemical reactions and 2) how the covariation and object aspects support students in linking the chemical context to mathematical representations.

1 INTRODUCTION

1.1 Mathematical Functions in Life Sciences

The application of mathematical functions is important in many STEM applications, as they are frequently used to describe real-world phenomena (Rogovchenko and Rogovchenko 2022). Therefore, a conceptual understanding of functions is a prerequisite for students entering STEM studies at university (Eberle et al. 2015; Neumann, Pigge, and Heinze 2017) even though many have problems dealing with them (Bain and Towns 2016; Ivanjek et al. 2016). One source of difficulty may be that functions, as an abstract concept, are only accessible through external representations (Duval 2006). The most common representations of functions are graphs, tables, algebraic equations and verbal descriptions. The latter can be considered as a situational description when a real-world context is given, e.g. every 20 minutes the number of cells doubles. To develop a sophisticated understanding of functions, one should be able to switch flexibly between these representations and know which is more appropriate for a given task. Research suggests that switching between representations is more difficult when a situational description is included (Bossé, Adu-Gyamfi, and Cheetham 2011), so it is expected that students will struggle to interpret graphs and the situational meaning of given parameters in algebraic equations.

Chemical kinetics, a subfield of physical chemistry, “is one of the areas that utilizes mathematics as its primary representation to communicate observations, analyses, and interpretations” (Bain and Towns 2016). It analyses the mechanisms of chemical reactions, focusing on reaction rates and the factors that influence those rates. This can be done by measuring the change in reactant concentration over time. The most basic chemical reactions are of zero and first order and can be described accordingly using linear or exponential functions (Elstner 2017). Several qualitative studies have reported the difficulties students have in interpreting graphs in chemical kinetics and in transferring knowledge from mathematics to chemistry and vice versa. Students tend to have a static view of the graph and associate it with specific phenomena, i.e. a Michaelis-Menten curve. This can be problematic when similar graphs occur in different applications and when a dynamic view of functions requires two or more varying quantities to be considered. In general, the graph covers a lot of information simultaneously in the context of chemical reactions. In fact, students tend to feel anxious when dealing with a graph, resulting in lower performance when asked equivalent questions using a graph instead of a table or algebraic equation (Rodriguez et al. 2019). In order for students to feel more confident in dealing with the mathematical representations and mapping contextual meaning onto these representations, they need to have a sophisticated conceptual understanding of functions. This paper explores how students engage with mathematical functions in a life sciences context, and how aspects of functional thinking might help to map contextual meaning onto mathematical representations.

1.2 Developing functional thinking

Functional thinking is “a way of thinking that is typical for dealing with functions” (Vollrath 1989). Three key aspects of functions are said to be typical: correspondence, covariation and the function as an object. The correspondence aspect emphasises that a function describes a relationship between two quantities, where each element of one quantity, e.g. the independent variable such as time, is mapped to an element of the other quantity, e.g., the dependent variable such as distance. Covariation focuses on how a change in the value of the independent variable affects changes in the value of the dependent variable. Viewing the function as a whole requires viewing the given functional relation as a new object that has its own properties and can be manipulated by operations. The correspondence aspect, which is mainly present in the definitions of functions, is the easiest for students to grasp. However, covariation is more important for understanding functional relations, and many difficulties students have can be explained by an inability to perceive the changing nature of functions (Malle 2000). Covariational reasoning is developmental and distinct levels of covariation can be achieved (Thompson and Carlson 2017). If someone can reason on a certain level, they are expected to be able to reason on all levels below that. Consequently, teaching the concept of function with a special focus on covariation should aim at the highest level which includes the ability to envision covariation smoothly and continuously.

Lichti investigated to what extent hands-on experiments or equivalent simulations with GeoGebra applets promote functional thinking (Lichti 2019). After the intervention, both groups showed significant learning gains in functional thinking with the simulation group making greater progress. Based on the students' written answers, the simulations promote reasoning based on covariation, while the hands-on group focused on the mapping of single values, i.e. correspondence. This is comprehensible due to the dynamic features of the GeoGebra applets, allowing for quick and easy manipulations of the independent variable to obtain values of the dependent variable instantaneously, thus enabling students to grasp the two covarying quantities more easily. Additionally, connections between different representations can be realised through subsidiary lines or colouring. The graph was found to be more efficient than a table for learning qualitative and quantitative functional thinking (Rolfes, Roth, and Schnotz 2022).

2 METHODOLOGY

2.1 Participants

The current study took place in the first semester of an undergraduate mathematics course at the School of Life Sciences of the University of Applied Sciences and Arts Northwestern Switzerland (FHNW). The course is compulsory for all undergraduates and therefore students from seven different fields of study take part. The learning environment was implemented during the first two weeks of the functions topic and consisted of consecutive exercises with corresponding GeoGebra applets. The material was provided through the MOODLE learning management system (LMS). After finishing an exercise, the students were asked to upload a file with their written answers. The analysed data refers to the second exercise of the learning environment. In total n= 89 students submitted their written answers of the second exercise.

2.2 Design of the learning environment

In developing the learning environment, the objectives were to identify a life sciences context 1) that is based on the concept of function and has the potential to promote conceptual knowledge and 2) that uses the representation of a graph with naturally arising varying quantities. These criteria were met by chemical kinetics, which has the potential to be beneficial for teaching the concept of functions (Rodriguez et al. 2019). As not all students choose a study direction that involves chemical kinetics, the learning environment should not require any prior knowledge of chemistry, but should leave some room for possible argumentation using prior chemistry knowledge.

With the development of GeoGebra applets, a chemical reaction was simulated and linked to a graph. The applets are therefore divided into two main parts, see Figure 1. The right part shows the developing graphs over time. The left part shows three bars representing the concentrations of the two reactants. The furthest left bar illustrates the cumulative concentration of both reactants, emphasising that 1) total concentration is constant over time and 2) the reaction could take place in a single vessel. The two adjacent bars show the concentration of each reactant separately. The height of the bars is related to the developing graphs. The dashed lines starting at the height of the bars and ending at the corresponding point in a two-dimensional coordinate system support the transfer from the simulated situation to the mathematical graph. To run the reaction, students can drag the *time* slider, which will simultaneously decrease concentration A and increase concentration B while the elapsed time is visualized by a thick black line on the horizontal axis. Below the coordinate system, check boxes allow the graph of each reactant to be shown or hidden.

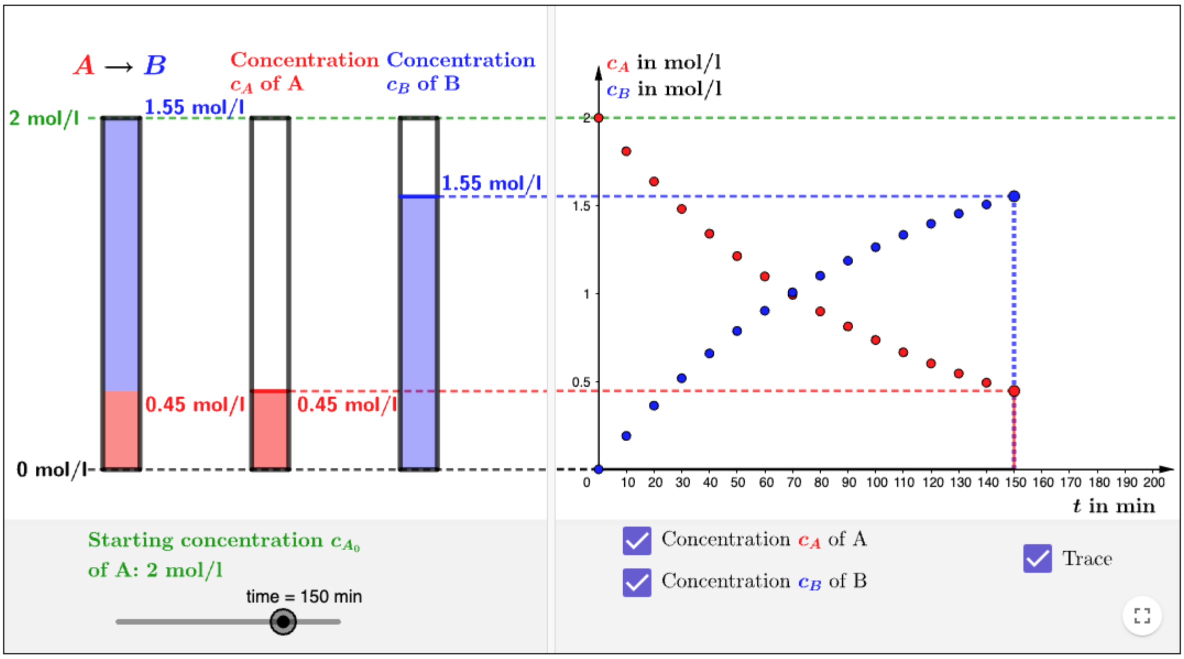

Fig. 1. Structure of the GeoGebra Applet

Activating the *trace* checkbox leaves all measured points, i.e. the potential curve, visible. With this applet students can easily carry out the chemical reaction and experience the connection to an evolving graph without doing any procedural tasks. The tasks in the exercises can therefore focus on describing and explaining the dynamic process of the chemical reaction in multiple representations and the translation between these representations. For example, in the second exercise the graph is introduced, and the first subtasks ask the students to explain how the elements of the bar chart (slider and height of the bars) are related to the moving point in the coordinate system, see Figure 2. Subsequent tasks require a comparison of the change in concentration during given time periods and an explanation why the change in concentration is continuously decreasing.

2.3 Data Analysis

To analyse the students' written answers, a content structuring qualitative analysis procedure was used (Kuckartz and Rädiker 2022). After the initial data overview, deductive main categories from literature were identified to code the material. The main categories consist of the three aspects of functional thinking, i.e. correspondence, covariation and object, as well as the different representations of functions used in the applet, i.e. graph, algebraic equation, bar chart and situational description. After the initial coding process, each main category is divided into smaller subcategories which show how students used the three aspects of functional thinking in the context of a chemical reaction. In addition, we can show how they were linked to different representations and how they might be helpful in the transition between representations. In particular, switching between a mathematical representation and the situational description is analysed. The analysed task consists of ten subtasks which have been coded separately. Multiple codes can be assigned per subtask or sentence, illustrated in Figure 2. Since the aspects of functional thinking can occur in combination with different representations the coded segments can overlap or intersect.

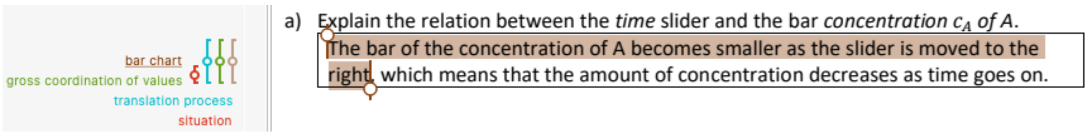

Fig. 2. Coding a student answer in MAXQDA

3 RESULTS

3.1 Subcategories of the aspects of functional thinking

The subcategories of the correspondence aspect are listed in Table 1. Due to difficulties in coding the students' responses without misinterpreting their thoughts, another category (mapping (vague)) has been added for answers that probably refer to the code "mapping". The last subcategory already indicates a dynamic view of the relationship between time and concentration and can be considered as a pre-category of covariation. As it explicitly focuses on the mapping of multiple time values to a concentration value, it still belongs to the correspondence aspect.

Table 1. Subcategories of the correspondence aspect

Subcategory	Description	Examples	Frequency
Mapping	A student writes that time is mapped to the concentration.	The red point shows the concentration of A at a certain point in time.	36
Mapping (vague)	A relationship between time and concentration is described. However, it is not clearly characterised as a mapping of the two quantities.	The height of the column indicates the concentration of A in the context of the minutes.	17
Starting concentration	The initial concentration of A (2 mol/l) or B (0 mol/l) at time t = 0 min is men-tioned. Both quantities must be given.	During this reaction, at time t=0, c_A of A is at 2 mol/l and c_B at 2 mol/l.	12
Mapping (dynamic)	Time is seen as a variable quantity. Nevertheless, the process of mapping time onto concentration is described.	As you drag slider t, bar A indicates the corresponding concentration.	4

Comparing frequencies of the correspondence aspect subcategories with those of the covariation aspect, summarised in Table 2, it becomes clear that students more often describe features of the applets that show the dynamically changing quantities. Given the levels of covariational reasoning, only the level of gross coordination of values could be clearly identified. One could argue that the distance or secant line between adjacent points indicates argumentation at a higher level, i.e. chunky continuous covariation, but it is not clear whether students imagine going through all these values between adjacent points or not. So, another label was chosen.

Table 2. Subcategories of the covariation aspect

Subcategory	Description	Example	Frequency
Quantitative	Students calculate the absolute change in concentration or describe the procedure.	Look at the position of the previous point on the graph. Read the y-value. Do the same for the next point, then calculate the difference.	31
Gross coordination of values (see (Thompson and Carlson 2017))	Students write that concentration increases or decreases as time increases.	The concentration decreases as the time increases.	132
Distance of points	The change in time and concentration is registered by the distance between adjacent points.	The points in the coordinate system in period 1 have a greater difference than those in period 2.	32
Slope	The change in time and concentration is described by the slope at a single point or of a secant between adjacent points.	If one were to connect the two points for the two periods, the slope would be greater for the first period → greater change	53

Overall, the frequencies of all subcategories of the object aspect, summarized in Table 3, are higher than those of the correspondence aspect but lower than those of the covariation aspect. Some subcategories of the object aspect implicitly describe the change in time and concentration, i.e. flattening, declining change or monotonicity. They were assigned to the object aspect because they describe properties of an exponential function that are only accessible if considering the whole function.

Table 3. Subcategories of the object aspect

Subcategory	Description	Example	Frequency
Flattening	The flattening graph is described.	The red curve flattens with time.	70
Function type	A function type, i. e. linear, exponential, or Michael-Menten curve is given to describe or delimit the functional relationship.	An exponential decrease can be detected across the measuring points.	71
Declining change	The change in concentration is detected as decreasing with time.	The more time passes, the less the concentration decreases.	28
Monotonicity	The graph or concen-tration of A or B is considered to be decreasing or increasing over the entire time period.	The coordinates show the progress of c_A, which decreases more and more and c_B, which increases more and more.	58

Obviously, if a person captures the declining decrease in concentration over time they must somehow imagine two covarying quantities, i.e. covariation. The category function type was coded most often, which is understandable as students should already know about polynomial and exponential functions. It remains open, whether they recognise the shape of the graph or other properties to identify a particular function type.

3.2 Representations and translation processes between mathematical representations and situation descriptions

In this exercise “graph” was coded $n_{FG} = 529$ times and “bar chart” was coded $n_{BC} = 80$ times. One person tried to derive an algebraic equation and no one made a table. Since these representations are not provided in the applet, this was to be expected. Graph and bar chart were coded, when the term itself or parts of each representation were explicitly mentioned. The bar chart and the graph were mentioned together in 56 cases, most of which consist of the description that the height of the bar describing the concentration of A is equal to the height of the red dot in the coordination system. This does not necessarily address one of the aspects of functional thinking. On the other hand, when only the bar chart is mentioned, most answers address a subcategory of the covariation aspect. This suggests that students who recognise the dynamic relationship between time and concentration in the bar chart, and see that the height of the bar is equal to the height of the point, are likely to see the graph as describing two changing quantities. We also identified translations between the mathematical representations (bar chart, graph) and the situation. As the applet covers some terms that can be used to describe the chemical reaction, we could not code every response that contained the words concentration or time. Therefore, only those responses were coded that clearly attempted to map contextual meaning onto the mathematical representations. Currently, 73 responses show a translation between the situation and either the graph or the bar chart. A subcategory of covariation is addressed in most of them. An example is: “The height of the bar indicates the corresponding concentration. If the concentration decreases, the red point decreases accordingly”. In the first sentence, the person imagines that the bar represents the concentration of A, i.e. a translation from the bar chart to the situation has occurred. The next sentence describes the change in concentration and explains the movement of the point, i.e. a translation was made starting from the situation to explain the movement of the point. Another example illustrates the translation from the graph to a situation description: “The red point in the coordinate system moves to the bottom right when the slider is moved to the right, which means that the concentration decreases over time”. Here the person describes the movement of the point covariationally (subcategory: gross coordination of values) followed by a situation description of the change in concentration A.

4 SUMMARY

Although some students seemed to have difficulties explaining the functional relationship between time and concentration in written form, the bar chart simulation firstly helped students to understand the dynamic relationship between time and concentration because they described the situation dynamically when referring only to the bar chart. Secondly, the height translation between the bar and the point in the coordinate system indicates that they relate the bar chart or the situational context to the graph and see the graph as a representation that describes a dynamic situation. Students tended to focus more on the covariation or object aspect than on the correspondence aspect, in line with literature (Vollrath 1989; Lichti 2019). Covariational reasoning (Thompson and Carlson 2017) was only present at the level of gross coordination of values, which could be due to the nature of the exercises, and it is possible that students were capable of higher level reasoning. Two reasons may explain the students’ focus on the object aspect: 1) the representation of a graph is suitable for recognising the whole function and 2) students use prior knowledge from school to help them recognise function types as well as typical properties of functions. In the next course, accompanying video material may be recorded to give more insight into the use of the applets and allow a better distinction between the aspects of functional thinking.

Overall, the study suggests that dynamic applets have high potential for visualising covariation features of functions, but additional exercises could encourage more elaborate covariational reasoning, i.e., higher levels of covariational reasoning. In addition, preliminary results suggest that covariational reasoning supports translation processes involving a situation description. To support a covariational view in teaching the concept of function, computer-based simulations offer great potential because the underlying relationships are presented dynamically. Instructors then need to focus on the process that leads to an entire graph, rather than just on the result. Analysing the change in incremental intervals by looking at adjacent points or by anticipating values between points is important for thinking about different types of function that might be appropriate to describe the relationship of the underlying quantities.

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