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Research paper

Novice and expert teachers' use of content-related knowledge during pedagogical reasoning

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ABSTRACT

We investigate the use of content knowledge and pedagogical content knowledge among 32 novices' and different types of experts' pedagogical reasoning, as exemplified in the field of mathematics education. Think-aloud interviews based on a planning task that required pedagogical reasoning were evaluated using verbal analysis. Our results show differences in the use of mathematics-related content knowledge and pedagogical content knowledge between novices and experts, as well as between types of experts. We conclude that novices tend to emphasise knowledge of teaching procedures and that experts' use of knowledge is related to their respective qualifications and experiences.

1. Introduction

Shulmans' (1986) distinction between *content knowledge* (CK) and *pedagogical content knowledge* (PCK) as components of teachers' professional knowledge is one of the most influential ideas in educational research. Since then, much research has been carried out with respect to the conceptualisations and measurement of CK and PCK, where PCK is often defined by knowledge subcomponents that it covers, such as knowledge of students' (mis) conceptions, curricular knowledge, or knowledge of teaching procedures (cf. Depaepe et al., 2013; Hume et al., 2019; Van Driel & Berry, 2010). Notably, recent research also explores teachers' *use* of CK and PCK in specific teaching situations in order to better understand the relationship between these knowledge components and practice (cf. Escudero & Sánchez, 2007; Koberstein-Schwarz and Meisert (2022); Krepf et al., 2018; Rieu et al., 2022; Tigelaar et al., 2017).

It appears particularly fruitful to investigate teachers' use of professional knowledge and how it influences practice by exploring teachers *pedagogical reasoning*, that is, the thinking that underpins their informed and professional practice (Loughran, 2019). Research on pedagogical reasoning assumes teaching to be much more than 'doing teaching'. Rather, teaching it is seen as teachers' constant choosing between alternative teaching procedures based on their professional knowledge (cf. Loughran, 2019; Loughran et al., 2016; Shulman, 1987). Consequently, the investigation of teachers' pedagogical reasoning allows for a deeper understanding of teachers' professional knowledge that is related and relevant to their practice.

In extant studies, *practical theories of teaching* have mainly been used to address and investigate teachers' private and experienced-based knowledge and beliefs that impact their pedagogical reasoning and practice (Cornett et al., 1990; Karabon, 2021; Kettle & Sellars, 1996; Levin et al., 2013; Levin & He, 2008; Tiilikainen et al., 2019). Less is known about the use of content-specific knowledge during pedagogical reasoning, such as CK and PCK. While Shulman (1986, 1987) was early to state that knowledge about instructional strategies and representations and knowledge about students' (mis)conceptions and difficulties are at the heart of PCK and pedagogical reasoning, few studies have empirically reconstructed the content-related knowledge components that teachers use during pedagogical reasoning or empirically investigated teachers' use of (predefined) content-related knowledge components during

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pedagogical reasoning. For instance, exceptions are Cunningham (2007) in the field of history education, Elliott (1996) in the field of language education, Loughran et al. (2004) in the field of science education, or Sánchez and Llinares (2003) in the field of mathematics education. However, because most research focuses on one or only a few content-related (sub-)components of professional knowledge, there is still no clear picture of how the various components are at work in pedagogical reasoning.

To address the question of which knowledge components teachers use during pedagogical reasoning, it is instrumental to conduct empirical studies with expert teachers and reconstruct their use of knowledge components. It is common in expertise research to contrast novices and experts in order to understand the role of knowledge that underpins professional practice (cf. Bromme, 1992; Glaser & Chi, 1988). To account for the diversity of expertise and its potential impact on the use of knowledge during pedagogical reasoning, it appears highly relevant in research to contrast the use of knowledge not only between novices and experts but also between different expertise groups (depending on qualifications and experiences). To our knowledge, there is no research that investigates these connections by systematically distinguishing between different content-related knowledge components and expertise profiles simultaneously.

We therefore investigate the use of CK and PCK that can be found with novices' and different types of experts' pedagogical reasoning, as exemplified in the field of mathematics education. It is our goal to provide empirical evidence that the predominant use of content-related knowledge components, such as knowledge of students' (mis)conceptions, curricular knowledge, or knowledge of teaching procedures, during pedagogical reasoning is related to teachers' prior qualifications and experiences. Consequently, there is likely not only a single approach regarding the use of content-related knowledge components in pedagogical reasoning but, rather, complementary approaches that focus on different components of content-related knowledge. Unraveling these approaches and their relationships to types of expertise may also provide insights relevant to promoting rich pedagogical reasoning in the context of teacher education; they may influence teacher educators cognitive modelling of lesson planning and teaching practices which has been found to be a significant mediator to the effect of their supervision to prospective teachers instructional skills (Mok & Staub, 2021). Although the context of our study is mathematics, we seek to contribute to a broader discussion by researchers in different school subjects and teacher education in general.

Conceptualisations of CK and PCK often refer to correct and institutionalised knowledge, which is assumed to be shared by the respective educational community. In this study, however, we use an existing conceptualisation of CK and PCK provided by Ball et al. (2008) but we do not assume that the knowledge must be correct from a normative point of view. We consider these components to also be comprised of incorrect, incomplete, subjective, or experience-based knowledge. Therefore, our approach combines research on professional CK and PCK with research on teachers' personal practical theories. This makes sense because prospective teachers still develop professional knowledge, and our study aims to contrast novice and expert teachers' use of content-related knowledge during pedagogical reasoning according to the frequencies with which CK and PCK are used (instead of focusing on the quality of CK and PCK).

In the following, we define components of mathematics-related knowledge, pedagogical reasoning, and types of experts in teaching mathematics for the purpose of our study. Moreover, we discuss differences between novices and types of experts and state our research question and hypotheses.

1.1. Professional teacher knowledge

Shulman (1987) argued that professional teachers do not simply perform trained scripts of behavior during teaching. Instead, professional teachers are said to rely on knowledge that is necessary to come to reasoned decisions according to the selection of teaching procedures in each given situation. Following Shulman, the conceptualisation of teachers' professional knowledge received much attention in educational research. Based on his seminal differentiation between CK and PCK (Shulman, 1986), much research has been carried out with respect to the (re-)conceptualisation and measurement of CK and PCK (cf. Hume et al., 2019; Van Driel & Berry, 2010), particularly in the field of mathematics education (cf. Ball et al., 2008; Blömeke et al., 2010; Depaepe et al., 2013; Hill et al., 2004; Krauss et al., 2008).

In particular, the conceptualisation of *mathematical knowledge for teaching* (MKT) by Ball et al. (2008) is among the most influential in mathematics educational research (Berry et al., 2016; Depaepe et al., 2013). On the one hand, MKT includes CK, which is further divided into the sub-components of *common content knowledge* (CCK, which refers to mathematical knowledge that is also relevant in settings other than teaching), *specialized content knowledge* (SCK, which refers to mathematical knowledge that is only relevant to mathematics teachers), and *horizon content knowledge* (HCK, which refers to mathematical knowledge about the connectedness of mathematical topics). On the other hand, MKT includes PCK, which is further divided into the sub-components of *knowledge of content and students* (KCS, which refers to knowledge about students' mathematical thinking or behavior), *knowledge of content and teaching* (KCT, which refers to knowledge about the design of mathematics-related instruction), and *knowledge of content and curriculum* (KCC, which refers to knowledge about mathematics-related instructional materials and teaching standards).

1.2. Pedagogical reasoning

In order to investigate the use of such different knowledge components for teaching, a framework is needed that specifies situations of knowledge use. In this study, we adapt the concept of pedagogical reasoning because it allows for a deeper understanding of teachers' knowledge and its relationship to their practice.

According to Loughran (2019), in contexts of practice, teachers essentially rely on knowledge about teaching procedures. A *teaching procedure* can be described as an activity by the teacher with the intention of transforming subject matter to make it comprehensible to students. In a more narrow sense, we describe a teaching procedure in Shulman's (1987) terms as a sequence of *representations*, such as ‘analogies, metaphors, examples, demonstrations, explanations, and so forth’ (p. 15), which can be used by the teacher to present the subject-matter to the students. Hence, in this context, the term “representation” is used to refer to an external (visible or tangible) production—as opposed to an internal (mental) construction (cf. Amador et al., 2022; Goldin, 2020)—that can support students' subject-specific learning (cf. Dreher et al., 2016; Erduran & Kaya, 2018).

In a given situation, however, teachers must often decide between multiple (more or less equally reasonable) alternatives of teaching procedures. In educational research, teachers' thinking according to the selection of teaching procedures is referred to as *pedagogical reasoning* (Kavanagh et al., 2020; Loughran, 2019; Loughran et al., 2016). In pedagogical reasoning, teachers are particularly supposed to apply content-related knowledge (e.g., of subject matter, students' mis-conceptions, or the curriculum) in order to attach teaching procedures to arguments that help to decide which teaching procedure is suitable in a given situation

(Loughran, 2019; Shulman, 1987). It can be considered to be a broadly accepted assumption that teachers' pedagogical reasoning is based on such knowledge.

Currently, it is subject to research how to study and foster the pedagogical reasoning ability of prospective teachers (cf. Kavanagh et al., 2020). Regarding the use of content-related knowledge during pedagogical reasoning, previous research has been carried out with (prospective) teachers across many different fields of education (Loughran et al., 2016). However, few studies have empirically reconstructed the knowledge components that teachers use during pedagogical reasoning or empirically investigated teachers' use of (predefined) knowledge components during pedagogical reasoning (see above). Within the field of mathematics education, empirical research has mainly focused on the question of how the pedagogical reasoning ability of (prospective) teachers can be supported by learning opportunities regarding the construction of task sequences (Andrews-Larsson et al., 2021) or regarding the planning of teaching (Kim et al., 2020), as well as how pedagogical reasoning ability is itself affected by prior subject matter knowledge (Sánchez & Llinares, 2003) or interactions among teacher colleagues (Horn, 2005, 2010). A few (rather early) studies have compared the mathematics-related pedagogical reasoning among different expertise groups (Borko & Livingston, 1989; von Minden & Walls, 1998). Despite the mentioned efforts in extant studies, it remains to current and future research to contrast the use of CK and PCK (and its various sub-components) between novices and varying expertise groups during pedagogical reasoning, particularly in the field of mathematics education.

1.3. Novices and experts

As outlined above, we seek to investigate the use of CK and PCK among novices and different expertise groups during pedagogical reasoning, as exemplified in the field of mathematics education. Who can be considered a novice or an expert? On the one hand, prospective teachers (in the first phase of teacher education) are often considered novices (Bromme, 1992). Prospective teachers have not completed their training in teacher education and have little experiences in teaching and pedagogical reasoning. On the other hand, it is not obvious who qualifies as an expert. According to Bromme (1992), experts cope with so-called knowledge-rich tasks, such as pedagogical reasoning, via the activation of professional knowledge. Because pedagogical reasoning is a reflexive activity, teachers who also serve as teacher educators are appropriate candidates to serve as experts. Teachers who also serve as teacher educators are the canonical counterpart to prospective teachers in the setting of teacher education. These experts can be assumed to have professional knowledge available and be experienced in teaching and pedagogical reasoning.

According to Murray et al. (2009), teacher educators differ with respect to their university degree (e.g., BA/MA in elementary or secondary education, MA in education, mathematics, or mathematics education), research (e.g., PhD in mathematics, mathematics education, or a related field), years of experience (in school and/or teacher education), and respective functions in teacher education programs. In teacher education, teachers with a BA/MA in elementary or secondary education and many years of teaching experiences are often assigned to supervise prospective teachers in the field. These teachers rarely have additional academic or mentoring qualifications and often do not see themselves as teacher educators (Feiman-Nemser, 1998). In the following, we call these teachers (who supervise mathematics teaching) *experts in teaching mathematics.* In the literature, they are also called school-based teacher educators or mentoring teachers. Moreover, there are teacher educators with a strong background in a subject or with a strong background in educational research (Murray et al., 2009). These teacher educators may also hold additional academic qualifications, such as a PhD in mathematics or mathematics education. We simply call them *experts in mathematics* and *experts in mathematics education.* Experts in mathematics often conduct subject-matter courses, whereas experts in mathematics education often conduct methods courses or research colloquia.

1.4. Differences between novices and expert groups

How do novices and the various experts compare with respect to the use of CK and PCK during pedagogical reasoning? Which of the knowledge sub-components of PCK differentiated by Ball and colleagues (see above) are likely to be emphasised by novices and the expert groups discussed above? Firstly, we refer to potential differences between experts in teaching mathematics, experts in mathematics, and experts in mathematics education (based on their respective qualifications and experiences). Secondly, we refer to potential differences between novices and experts in general.

Experts in mathematics teaching likely have much more teaching experience than the other experts. According to Shulman (1986), they must have at hand a rich repertoire of teaching procedures. As a consequence, these experts may routinely refer to refined teaching procedures (KCT) and experience-based knowledge about students' (mis-)conceptions and typical errors (KCS). Due to their well-reflected experiences, CK, KCS, and KCC have already been used during past reflections regarding the use of teaching procedures. Therefore, these experts' selection of teaching procedures in a given situation can be considered a 'sub-conscious process' (Loughran, 2019, p. 530). Hence, compared to the other groups of experts, it is expected that experts in mathematics teaching will tend to use more KCT when confronted with tasks that require pedagogical reasoning, such as lesson planning.

Experts in mathematics have extensive CK in mathematics but fewer teaching experiences (Murray et al., 2009). However, these experts are also experienced in pedagogical reasoning. Therefore, these experts likely arrive at decisions with respect to the use of teaching procedures mainly via the use of CK and further knowledge from the KCS or KCC component. Compared to other groups of experts, it is reasonable to assume that experts in mathematics will use more CK in pedagogical reasoning. This assumption is in line with Watson and Barton (2011), who argue that these experts mainly enact mathematics in planning and teaching situations.

Experts in mathematics education are literate in the field of mathematics education as a research discipline but also have fewer teaching experiences (Murray et al., 2010). Possibly, these experts also arrive at decisions with respect to the use of teaching procedures via the use of various research-based knowledge about teaching. Research in mathematics education is concerned with various aspects of learning and teaching mathematics. Particularly, experts in mathematics education are interested or involved in the research-based reconstruction of students' mathematics-related (mis)conceptions and typical errors and the research-based construction of teaching standards, textbooks and other materials designed to support mathematics teachers. Based on this, it can be assumed that experts in mathematics education more often emphasise KCC and KCS during pedagogical reasoning as compared to the other groups of experts. Because experts in mathematics teaching are also experienced in dealing with students (mis-)conceptions and typical errors, however, the assumption must be weakened regarding the use of KCS.

Because novices have little experience in pedagogical reasoning, they may be more likely to use KCT in reaction to tasks that require pedagogical reasoning instead of providing arguments based on further knowledge. This assumption is in line with Nilsson (2009), who reports that novices are often interested in practical knowledge (e.g., about teaching procedures). Nilsson also argues that more theoretical knowledge (such as CCK or KCC) is often experienced as not useful by novices and they tend to focus on doing rather than pedagogical reasoning. In summary, for novices, it can be assumed that they predominantly display KCT in reaction to tasks that require pedagogical reasoning.

With respect to the use of KCT, the difference between novices and experts in teaching mathematics may be smaller than the difference between novices and experts in mathematics or the difference between novices and experts in mathematics education. However, this does not mean that novices and experts in teaching mathematics are similar. The reasons why novices and experts in teaching mathematics use KCT is another: experts in teaching mathematics refer to well-refined teaching procedures (Loughran, 2019), whereas novices assumably do not see the necessity of providing arguments for or against the use of teaching procedures based on further knowledge (Nilsson, 2009). Consequentially, the quality of their KCT can be expected to show large differences.

In summary, it can be assumed that the predominant use of knowledge components during pedagogical reasoning is dependent on experts' qualifications and experiences. Moreover, novices tend to focus on doing rather than pedagogical reasoning. Therefore, they may predominantly display KCT in reaction to tasks that require pedagogical reasoning.

1.5. Research question and hypotheses

To state our research question and hypotheses, we build on the MKT conceptualisation of Ball et al. (2008) and distinguish between CK, KCS, KCT, and KCC. Because it is not always easy to distinguish between SCK and other sub-components within the MKT conceptualisation, both on a theoretical level (Petrou & Goulding, 2011) and on an empirical level (Copur-Gencturk & Tolar, 2022), and to avoid corresponding difficulties during data analysis, we do not explicitly distinguish between the sub-components of CCK, SCK and HCK within this study.

The conceptualisation of Ball and colleagues focuses on the knowledge that teachers need to know in order to carry out the work of teaching mathematics successfully. In this study, we use the conceptualisation to refer to different components of teachers' knowledge; however, we do not assume the knowledge (sub-) components to be restricted to knowledge that is correct from a normative point of view. We consider these components to also be comprised of incorrect, incomplete, subjective, or experience-based knowledge. Therefore, our approach combines research on professional CK and PCK with research on teachers' personal practical theories.[1] This procedure makes sense because prospective teachers still develop professional knowledge. Moreover, KCT is possibly often acquired on the basis of experience. In addition, it is scarcely possible to label the (intended) use of teaching procedures in a situation as clearly appropriate or not. Consequently, we do not study the quality of knowledge used by novices or experts. Rather, we seek to investigate what group uses *what* knowledge component more or less often compared to other groups.

Based on the mentioned considerations we are now able to pose our question of research: *How do novice teachers, experts in mathematics teaching, experts in mathematics and experts in mathematics education differ with respect to the predominant use of knowledge components (CK, KCS, KCT and KCC) during pedagogical reasoning?*

According to the expert approach by Bromme (1992), experts' use of knowledge in activities such as pedagogical reasoning can be related to their specific knowledge. Therefore, we derive hypotheses about differences between the expert groups based on their prior qualifications and experiences (H1–H3, see below), as discussed in the previous section.

To formulate our hypothesis concerning the differences between novices and experts in general, it is not possible to start with novices' prior qualifications and experiences. They have no relevant qualifications and hardly any teaching experience, so their use of specific knowledge components cannot be related to particular qualifications and experiences. Because novices presumably do not see the necessity of providing arguments for or against the use of teaching procedures based on content knowledge, student-related knowledge, or curricular knowledge (as discussed in the previous section), however, there is justification for the hypothesis that, compared to experts in general, novices predominantly display KCT in response to tasks that require pedagogical reasoning (H4, see below).

H1. Experts in mathematics teaching use more KCT than experts in mathematics and experts in mathematics education during pedagogical reasoning.

H2. Experts in mathematics use more CK than experts in mathematics teaching and experts in mathematics education during pedagogical reasoning.

H3. Experts in mathematics education use more KCC and KCS than experts in mathematics teaching and experts in mathematics during pedagogical reasoning.

H4. The novices use more KCT than the expert groups.

2. Method

In this study, we collected interview data in the form of think-aloud protocols in reaction to a task that requires pedagogical reasoning. The data are analysed with respect to the use of CK, KCS, KCT, and KCC by means of verbal analysis (Chi, 1997). Verbal analysis also entails the quantification of qualitative codings, which can be used to support inferences regarding the acceptance or rejection of our hypotheses, as stated above.

2.1. Design

In contexts of practice, teachers' pedagogical reasoning and use of knowledge is mental work and thus unobservable (Loughran, 2019). Hence, for these constructs, any method of data acquisition is confronted with the challenge of providing observable indicators of a construct that is principally unobservable. The use of CK, KCS, KCT, and KCC during pedagogical reasoning can hardly be captured via an instrument that 'measures' the respective use of knowledge components directly by providing quantitative data. Rather, we assume it to be appropriate to collect data by means of individual verbal responses to suitable questions used as stimuli. Subsequently, these data can be analysed with respect to the research question. In this study, think-aloud interviews are used so that research participants can verbalise their pedagogical reasoning. According to Leighton (2017), think-aloud interviews are adequate to providing the means to identify mental processes, participants pedagogical reasoning in this case.

[1] Cornett et al. (1990), for instance, defined practical theories of teaching as teachers' claims to knowledge about teaching practice that are based on experiences (see also Introduction).

Table 1
Study participants.

	Novices	Experts in teaching mathematics	Experts in mathematics education	Experts in mathematics
Sample size	12	7	6	7
Assigned sex	Female (9), male (3)	Female (5), male (2)	Female (3), male (3)	Female (5), male (2)
Mean age (SD)	21.33 (4.35)	45.71 (8.05)	50.33 (9.68)	51.14 (7.73)
Teaching certificate	None	Primary education	Primary education	Secondary education
Teaching experience	None	More than five years (three with more than 10 years)	More than five years	More than five years
Highest academic qualification (of all participants in the subgroup)	None	BA (primary education)	MA (mathematics education or related field)	PhD (mathematics or physics)

2.2. Participants

We consider prospective primary teachers in the first phase of teacher education to be *novices*. We chose to focus on prospective teachers in the first phase of teacher education because they have not completed their training in subject-matter education and have few experiences in teaching and pedagogical reasoning. The novices who participated in our study were all Bachelor's students in the first year of preparation within a teacher education program in the German-speaking part of Switzerland. With respect to mathematics education, participants had completed at least one course and a maximum of two out of four content-related courses (mathematics and mathematics education). Participants had not entered the practical training phase yet.

According to the experts, we use prior qualifications to differentiate between three groups with different types of expertise. (1) *Experts in teaching mathematics* are experienced mathematics teachers in primary classes (more than 10 years) who obtained a teaching certificate (typically a BA in primary education) and received some further training in order to serve as a practice mentor for prospective primary teachers during teacher education. (2) *Experts in mathematics education* are experienced primary mathematics teachers (more than 5 years) who obtained a teaching certificate and a university degree in mathematics education (MA or PhD); serve as a lecturer in a teacher education program; and are involved in the construction of teaching standards, textbooks, or other materials that are designed to support mathematics teachers. (3) *Experts in mathematics* are experienced mathematics teachers (more than 5 years) who obtained a teaching certificate (regardless of primary or secondary education) and a PhD in mathematics and serve as lecturers in teacher education programs.

The data were derived from 32 participants: twelve participants were novices and 20 participants were experts (seven experts in teaching mathematics, six experts in mathematics education, and seven experts in mathematics). Given the large effort that the verbal analysis entails, the sample represents a compromise so as to obtain a manageable amount of data.

All subjects voluntarily participated in the study. We sent invitation e-mails to a large number of potential candidates. The selection of participants took place in the order in which they responded to our e-mail. Additionally, we sought to balance the sample with respect to a) the expert group sizes and b) the novices' outcomes on a previous examination in mathematics and motivation to teach mathematics. Regarding the experts, we scanned the websites of schools of education in the German-speaking part of Switzerland in order to identify eligible persons or contact persons.

Due to the low response rates, we included three persons in the group of experts in teaching mathematics who did not meet all the conditions. These persons were experienced mathematics teachers in primary classes for more than five years (instead of more than 10 years of teaching experience; all other conditions apply). Moreover, one person in the group of experts in mathematics held a PhD in physics instead of mathematics (all other conditions apply).

In addition to Table 1 (and the information presented above), the following details on the subjects are relevant. Among the novices, six participants had completed one of four content-related courses in the teacher education programme. The other six novices had completed two courses. In the teacher education programme, five novices showed satisfactory outcomes on a previous mathematics examination on content knowledge (in Switzerland grade 4), four showed good outcomes (grade 5), and three showed very good outcomes (grade 6). Among the seven experts in teaching mathematics were two participants with further qualifications (one with an MA in special education and one with an MA in mathematics education).

2.3. Interviews

In line with related literature (e.g., Kim et al., 2020; Shulman, 1987), we assume that studying teachers' lesson planning helps to illuminate their pedagogical reasoning. In particular, the selection of subject-specific teaching procedures is important during lesson planning (see the Introduction). For instance, during planning, teachers need to select among (sequences of) mathematical tasks that are intended to be used during the lesson. Compared to teaching in vivo, the planning situation can also be better captured in the laboratory situation of a think-aloud interview (which is needed to investigate teachers' thought processes). Nonetheless, the task of planning is practice oriented and authentic to prospective teachers.

In the interviews, hence, the participants were confronted with the task of planning a lesson introducing the subtraction algorithm (written subtraction) to 4th graders. We chose this particular task for several reasons: (1) An introductory lesson on a new concept or procedure requires participants' special attention with respect to choosing appropriate teaching procedures. (2) The subtraction algorithm provides a good opportunity for the use of CK (subtraction with regrouping, for instance, is a non-trivial case to many teachers, cf. Ma, 1999), KCS (students' errors or conceptions are well documented in the literature, cf. Fiori & Zuccheri, 2005; Kühnhold & Padberg, 1986), KCT (the subtraction algorithm is a traditional topic in teaching mathematics in Switzerland and it is dealt with in lessons at several class levels), and KCC (it is dealt with in teaching standards and mathematics textbooks in Switzerland).[2]

[2] Moreover, the meaning of algorithms is regularly subject to scholarly debate in the mathematics education community in Switzerland. Currently, written multiplication and division has been removed from teaching standards in primary classes to reinforce the importance of semi-scriptural procedures to multiplication and division.

2.4. Procedure

All participants were asked to think out loud for approximately 15 min, that is, to verbalise everything that came to their mind with respect to the selection and arrangement of teaching procedures within that fictive lesson. This time restriction is chosen to trigger spontaneous reactions, simulate a familiar situation in which decisions must be made under time pressure, and produce a manageable amount of data. It was made clear to the participants that it was not important to finish the task. It was also communicated that the interview is not a conversation between the investigator and the participant but, rather, a laboratory situation in which the investigator only serves as an initiator of verbalised mental processes by providing the tasks. Only if a participant stopped thinking aloud did the investigator ask the participant to proceed thinking aloud ("What else comes to your mind according to the selection of teaching procedures?"). In some cases, participants did not know what else to think about. These participants were asked to refer to anything that comes in their mind according to an excerpt from a textbook, which represented the subtraction algorithm, that they had to use in the fictive lesson.

The procedure described above is based on criteria proposed by the think-aloud literature (Leigthon, 2017). An important aim of this method is to influence the interviewee's thinking as little as possible. Apart from the stimulus provided by the interviewer ('What comes to your mind with respect to the selection and arrangement of teaching procedures within that fictive lesson?'), there are thus no further content-related questions in the interview. According to Leigthon (2017), think-aloud interviews are an adequate means of identifying mental processes – in this case, the participants' pedagogical reasoning.

2.5. Analysis

As outlined above, the analysis focused on verbal data because they were judged to be the most appropriate for investigating the use of knowledge components during pedagogical reasoning. It was largely based on verbal analysis (Chi, 1997), which integrates qualitative content analysis and quantitative analysis. Therefore, it can be considered a type of mixed method, according to Kelle (2019). We pursued the following basic strategy. After transcribing the interviews, we segmented all interview protocols into proposition-sized units (first step). Thereafter, we coded each segment in each interview according to our coding scheme (second step). Essentially, our coding scheme reflects an operationalisation of CK, KCS, KCT, and KCC. Both steps (segmenting and coding) were carried out by two researchers. Their training and the development of the coding scheme took place in a pilot study that we conducted in advance. In the following two subsections, we present both steps of analysis in more detail, along with the inter-rater reliability. In the subsequent subsection, we describe the quantification of our coding and the related statistical analysis (the third step of analysis).

2.5.1. Segmenting the interviews

The segmentation of data is intended to obtain proposition-sized units (Chi, 1997). To achieve high inter-rater reliability according to the segmentation of the data, we applied a set of non-content rules instead of semantic evaluation. For instance, if two main clauses or subordinate clauses are connected with an "and," they were separated. Moreover, these rules referred to the separation of relative clauses and other compounds such as causal clauses. The resulting phrases are considered to represent the segments. Twenty percent of all interviews were segmented by two researchers. To measure the inter-rater reliability, we calculated the normalised Levensthein distance (nD) according to Kolbe et al. (2016): $nD = .09$. These authors suggest good reliability if $nD < 0.1$, which means that less than ten percent of segments had to be corrected.

2.5.2. Operationalisation of knowledge components

Ball et al. (2008) define their knowledge (sub-)components rather implicitly, as knowledge resources that underlie certain tasks. For instance, KCS is said to underlie the recognising of common errors. At the same time, however, SCK is said to underly the analysis of common errors, and CCK is said to underlie the recognition of wrong answers (ibid.). For this reason, in the pilot study it turned out to be difficult to adopt the MKT components as categories for analysis if one seeks to assign these categories to segments of transcribed interview data. Initially, we decided not to distinguish between the sub-components of content knowledge within this study. Secondly, we further operationalised the components of CK, KCS, KCT, and KCC for the purpose of our study.

We operationalise CK as a set of propositions that can, in principle, be justified by mathematical argumentation or definition; KCS as a set of propositions that can, in principle, be justified by the (ordinary or scientific) observation of students' behavior with respect to engagement in mathematical tasks, and KCC as a set of propositions that can, in principle, be justified by comparison with mathematical teaching programs/standards, textbooks, and related materials. According to these operationalisations, CK, KCS, and KCC are considered to comprise the respective propositional knowledge. In contrast to these definitions, KCT is operationalised as (verbalised) script knowledge (Anderson, 2007) about teaching procedures. With 'teaching procedures', we refer to any (temporal sequences of) representations (e.g., examples, pictures, questions, explanations, and tasks) that can potentially be used in the classroom with the intention of making mathematical concepts accessible to students. Ball et al. (2008) do not use the term 'teaching procedure' but also refer to (temporal sequences of) representations for mathematics teaching. From the definitions, it becomes clear that all knowledge components cover subjective knowledge, which can appear as incomplete or incorrect from a normative point of view and which can be based on experiences.

Our operationalisation of CK, KCS, KCT, and KCC essentially represents the coding scheme used to assign codes to the segments in interview protocols. In the coding process, we decided, for each segment in the interviews, whether the proposition contained in the segment could be evaluated by the differential criteria given above (and assigned the code CK, KCS, or KCC to the segment), whether it represents a teaching procedure (and assigned the code KCT), or whether none of these cases apply (and assigned the code OTHER). Table 2 provides examples of propositions and the assigning of codes.

Two researchers coded all interviews independently of one another. Overall, substantial inter-rater reliability (Cohens Kappa, k) was achieved according to Landis and Koch (1977): $k = 0.81$. Good reliability with respect to the coding process also indicates good validity on the part of the segmenting process.

2.5.3. Statistical analysis

We used the open-source software and programming language R (Version 4.1.1) for the statistical analysis of code frequencies. The frequency of a certain code in an interview is considered to be a measure of the use of the respective knowledge component during pedagogical reasoning in the interview. Alternatively, it is also possible to consider the relative frequency of a certain code in an interview (relativised to the number of segments in the interview). Due to the design of data acquisition, we expect the length of interviews (time and/or number of segments per interview) to be

Table 2
Example propositions and assigned codes.

Proposition	Assigned Code
"If the minuend is smaller than the subtrahend, the result is a negative number."	CK
"After the number 9, there is the next value unit."	CK
"A lot of students do not understand the decomposing of a higher value unit."	KCS
"In game situations, students use rather semi-scriptural approaches to subtraction."	KCS
"I let the students solve subtraction tasks using the Dienes material."	KCT
"Next, I will present students' different approaches to subtraction by using the visualizer."	KCT
"In textbooks, written subtraction is required in grade 3."	KCC
"Semi-scriptural approaches to subtraction play an important role in the textbook."	KCC
"I don't know what else to say."	OTHER
"Okay, you ask me to plan an introductory lesson to the subtraction algorithm."	OTHER

comparable. Nonetheless, we decided to analyse both the frequencies and relative frequencies to account for the potential influence of the interview length on the frequencies of the codes in an interview.

All dependent variables were tested for a normal distribution (both Lillifors- and Shapiro-Francia-Test) and the homogeneity of variances (Bartlett-Test) in all groups. Whenever possible, we applied the directed *t*-test to test our hypotheses H1—H3. If a normal distribution was not given, we applied the directed Wilcoxon-Test (Mann-Whitney-U-Test). We applied omnibus-testing according to H4: we used the Kruskal-Wallis-Test (because a normal distribution was not given and, hence, analysis of variances was not an option) and we performed post-hoc analysis using Dunn's Bonferroni-Test.

3. Results

As expected, due to the design of data acquisition, the length of interview protocols is quite comparable. Within our population ($n = 32$), the mean of interview length is 13 min 33 s, and the standard deviation is 2 min 77 s. In terms of the number of segments in an interview protocol, the mean interview length is 119.4 segments, and the standard deviation is 31.79 segments. Moreover, there seems to be no substantial difference between the groups. For instance, the mean length in terms of time is only slightly longer in the interviews with novices ($n = 12$, $M = 14{:}03$ min) as compared to the interviews with experts ($n = 20$, $M = 13{:}14$ min). The same holds with respect to length in terms of the number of segments in an interview protocol.

The frequency of a certain code in an interview can be interpreted as a measure of a participants' use of the respective knowledge component. Because the lengths of interviews is comparable, it is possible to compare participants' code frequencies (of CK, KCS, KCT, KCC, and OTHER) directly. Because we are interested in comparing groups (novices, experts in teaching mathematics, experts in mathematics education, and experts in mathematics) rather than individuals, we show means of code frequencies per group in the descriptive statistics (see Table 3). To account for the potential influence of the interview length on the frequencies of the codes in an interview, we nevertheless also analysed the mean values of the relative frequencies of the codes (relativised to the number of segments in an interview). Because all significant differences in the mean frequencies reported below can be reproduced by analysing mean relative frequencies, however, we refrain from presenting these additional results.

Hypothesis H1. According to H1, experts in mathematics teaching use more KCT than experts in mathematics and experts in mathematics education during pedagogical reasoning. The directed Wilcoxon-Test shows that experts in mathematics teaching do not use significantly more KCT than experts in mathematics education: $W(11) = 17$, $p = .31$. The effect size (Pearson's r) is $r = 0.16$, which can be considered a weak effect according to Cohen (1992). However, experts in mathematics teaching use significantly more KCT than experts in mathematics: $W(12) = 10$, $p = .04$. The effect size is $r = 0.49$, which can be considered a strong effect according to Cohen (1992).

Hypothesis H2. According to H2, experts in mathematics use more CK than experts in mathematics teaching and experts in mathematics education during pedagogical reasoning. The directed *t*-test test shows that experts in mathematics use significantly more CK than experts in mathematics education, $t(11) = 2.96$, $p = .007$. The effect size is $r = 0.67$, which can be considered a strong effect according to Cohen (1992). Similarly, experts in mathematics use significantly more CK than experts in teaching mathematics: $t(12) = 4.66$, $p = .0006$. The effect size is $r = 0.80$, which can be considered a strong effect according to Cohen (1992).

Hypothesis H3. According to H3, experts in mathematics education use more KCC and more KCS than experts in mathematics teaching and experts in mathematics during pedagogical reasoning. The directed Wilcoxon-Test shows that experts in mathematics education do not use significantly more KCC than experts in mathematics: $W(11) = 13.5$, $p = .15$. The effect size is $r = 0.30$, which can be considered a moderate effect according to Cohen (1992). Moreover, experts in mathematics education do not use significantly more KCC than experts in mathematics teaching: $W(11) = 32$, $p = .06$. However, the p-value is close to the border of significance. The effect size is $r = .44$, which can be considered a strong effect according to Cohen (1992). With respect to KCS, we refrain from further statistical analysis because it becomes evident from the descriptive statistics in Table 3 that all groups are quiet comparable.

Table 3
Means of frequencies of codes, standard deviations in brackets.

	Novices	Experts in mathematics teaching	Experts in mathematics education	Experts in mathematics
Mean frequency of code CK	15.5 (12.65)	5.00 (4.69)	10.83 (5.27)	22.57 (8.81)
Mean frequency of code KCS	19.83 (7.67)	18.29 (9.78)	18.50 (5.05)	18.71 (5.77)
Mean frequency of code KCT	76.25 (25.03)	62.14 (21.54)	60.5 (24.01)	37.43 (14.10)
Mean frequency of code KCC	2.91 (3.15)	4.29 (5.22)	8.67 (7.17)	4.86 (3.24)
Mean frequency of code OTHER	18.17 (9.82)	23.57 (18.44)	18.16 (11.29)	21.57 (17.27)

Hypothesis H4. According to H4, novices use more KCT than the expert groups. Omnibus testing (a Kruskal-Wallis-Test) shows that the novices and expert groups differ significantly with respect to the use of KCT: Chi-squared = 9.83, p = .02. However, the post-hoc analysis (Dunn-Bonferroni-Tests) reveals a more differentiated view. Only the difference between novices and experts in mathematics is revealed to be significant ($z = -3.10$, $p = .006$), with a strong effect (Cohen's d) according to Cohen (1992): d = 1.78.

4. Discussion

During pedagogical reasoning, teachers are also supposed to apply content-related knowledge in order to decide which teaching procedure is suitable in a given situation. In this study, we hypothesised that the use of knowledge during pedagogical reasoning differs between novices and groups of varying expertise. Essentially, all descriptive statistics comply with our hypotheses. Hence, the overall impression supports the validity of our analysis. In the following, we first summarise and classify our findings. Secondly, we discuss the support for our hypotheses in more detail. Thirdly, we discuss several limitations of our study. Finally, we conclude by discussing the implications for further research and teacher education.

4.1. Summary and classification of results

We applied inference statistical analysis to check for significant differences between the groups. According to H1, experts in teaching mathematics use more KCT in pedagogical reasoning than experts in mathematics education and experts in mathematics. Based on our data and statistical inferences, this hypothesis is partially supported. Our results suggest that experts in teaching mathematics use more KCT than experts in mathematics, whereas experts in teaching mathematics and mathematics education are rather similar. According to H2, experts in mathematics use more CK than experts in mathematics education and experts in teaching mathematics. This hypothesis is fully supported based on our data and statistical inferences. This finding is in line with Watson and Barton (2011), who argue that mathematics experts mainly enact mathematics in planning and teaching situations. However, we were not able to find sufficient support for H3, which held that experts in mathematics education use more KCC and KCS in pedagogical reasoning as compared to experts in mathematics and experts in teaching mathematics. Finally, we found some evidence supporting H4: there seems to be a difference between novices and experts regarding the use of KCT, particularly between novices and experts in mathematics.

Overall, our findings support the view that KCS could be equally important for pedagogical reasoning in all expert groups. However, the use of CK, KCT, and partially also KCC differs between the expert groups. Consequentially, these knowledge components are not generally equally important during pedagogical reasoning but, rather, related to teachers' qualifications and experiences. Compared to expert groups, moreover, novices tend to emphasise KCT in reaction to tasks that require pedagogical reasoning. This finding is in line with the assumption that novices have little experience in pedagogical reasoning and are more interested in practical knowledge about teaching procedures than ‘theoretical’ knowledge such as CK or KCC (cf. Nilsson, 2009).

In summary, the findings of our study demonstrate the various ways in which experts' and novices' use of knowledge components unfolds. Based on our analysis, it is likely that there is not only ‘one best road’ regarding the use of experts' knowledge components during pedagogical reasoning but, rather, complementary strategies that emphasise different knowledge components. With respect to novices, our findings suggest that it must be considered a challenge for novices to prioritise the role of pedagogical reasoning and the related use of various knowledge components, instead of visible teaching behavior.

Taking these findings into account, we believe our research represents a promising attempt to bring together different strands of research: pedagogical reasoning (via the framework that we used to specify situations of knowledge use), conceptualizations of content-related knowledge (via the conceptualisation that we used to identify the various components of content-related knowledge), and, finally, practical theories of teaching (via taking into account also subjective or experiece-based knowlegde in order acknowledge novice and experts individual knowledge relevant to their practice).

4.2. Support of hypotheses

Regarding H1, our data may have been influenced by the fact that the expertise profile in mathematics education is close to the expertise profile in teaching mathematics: both groups have teaching experience in primary classes. Moreover, experts in mathematics education were involved in the construction of teaching standards, textbooks, or other materials. Hence, they were likely oriented towards the practical and constructive dimension of mathematics education research. Because all groups deployed a relatively large amount of KCT in pedagogical reasoning (see Table 3), it is also possible that the task used in the interviews did not maximise the potential to discriminate in terms of the use of KCT during pedagogical reasoning.

Regarding H2, a strong background in mathematics seems to be a feature that contributes a great deal to the use of CK during pedagogical reasoning. Although the subtraction algorithm may appear to be a trivial procedure to experts in mathematics and the challenge to these experts is, rather, how to present it to students, they deployed much more CK during pedagogical reasoning as compared to the other groups. It seems likely that these experts found it important to reflect on the mathematical concepts and procedures associated with the subtraction algorithm. Moreover, these experts may also compensate for missing KCC or KCT by relying on CK in order to come to decisions with respect to the use of teaching procedures. For this reason, strong effect sizes ($r = 0.67$ resp. $r = 0.8$) may also refer the fact that experts in mathematics had teaching experiences in secondary classes instead of primary classes.

Regarding H3 and the use of KCC, the effects are moderate, and the p-value is close to significance in one of the two comparisons. Thus, with more statistical power, it could be possible to obtain significant results. Regarding the use of KCS, no differences between the expert groups were observed. This also comports with the initial appraisal that it is difficult to determine which of the groups emphasise KCS (see the Introduction).

Regarding H4, we find support for the notion that novices use more KCT than the expert groups. However, only novices and experts in mathematics differ significantly in this regard. Nonetheless, it is impressive that novices use even more KCT than experts in teaching mathematics, according to the descriptive statistics. Our results are interesting because both novices and experts in mathematics have little or no teaching experience in teaching mathematics in primary classes. In contrast to the situation for experts in mathematics, this does not hinder novices from drawing heavily on KCT.

4.3. Limitations

Obviously, the participants' use of knowledge components during pedagogical reasoning is influenced by the task used in the interview. Because all participants were confronted with the same task, however, we believe it is possible to compare the use of knowledge between the groups (interindividual differences). As Bromme (1992) argues, however, a task may appear differently to novices and experts. Therefore, it may be helpful in future research to triangulate the use of tasks.

Note that we are generally not able to reject hypotheses based on our data. Due to our small sample, it is only possibly to confirm hypotheses when there are (very) strong effect sizes. Finding support for hypotheses in case of moderate or low effect sizes would require larger sample sizes. In future research, our hypothesis may be adapted so as to be tested with a larger sample.

Furthermore, the knowledge components are different with respect to their levels of abstraction. For instance, the KCT component is operationalised in a way that many more segments in the interviews can be assigned to this component. Compared to KCT, for instance, the KCC component is much more specific and, for this reason, appears much less frequently during analysis. Because the frequency of knowledge components is dependent on both the task used in the interview and their level of abstraction, it is not possible to analyse the columns in Table 3 (intraindividual differences) but, rather, only its lines (interindividual differences). For instance, the fact that novices use more KCT than KCC in our study could be an artefact of the task used in the interview and the different levels of abstraction of KCT and KCC.

4.4. Outlook

As outlined above, more research with larger samples is required to stabilise and explain our results. In future research, it seems to be important to also include qualitative factors in order to come to a better understanding of the findings presented in this study, for instance, with respect to the quality of KCT displayed by novices and experts in teaching mathematics or with respect to describing the use of knowledge components in argumentation processes. Regarding teacher education programs, we conclude that it seems to be important for novices to learn how to come to decisions regarding the use of teaching procedures via the application of further content-related knowledge components. In teacher education, it could be promising to not only create opportunities for novices to practice engaging in pedagogical reasoning but also highlight the use of relevant knowledge from CK, KCS, and KCC needed in pedagogical reasoning (Mok & Staub, 2021). Kavanagh et al. (2020), for instance, reports on teacher educators who offered guidance to novices regarding how to engage with relevant knowledge components during pedagogical reasoning. In order to provide the grounds on which to promote rich pedagogical reasoning that is based on content-related knowledge, however, there is a need for more research on (the training of) content-related pedagogical reasoning.

Declaration of competing interest

The authors declare that they have no conflict of interest. All authors have approved the final article.

Data availability

Data will be made available on request.

Appendix A

Table A1
List of abbreviations defined in the Introduction

Acronym	Definition
CK	Content knowledge
CCK	Common content knowledge
HCK	Horizon content knowledge
PCK	Pedagogical content knowledge
KCC	Knowledge of content and curriculum
KCS	Knowledge of content and students
KCT	Knowledge of content and teaching
MKT	Mathematical knowledge for teaching
SCK	Specialized content knowledge

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