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Entry

Vincenzo Galilei and Musical Experiments

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Definition

There is no consensus among historians when it comes to the importance of Vincenzo Galilei's role in the history of music and science, especially when it comes to his contribution to the birth of modern experimentalism. Galilei's written works, even those left in manuscript form, most of which have now been transcribed and published, do not provide a clear picture of his contribution. Moreover, there is a lack of private documents, such as letters, which informally describe his approach, working hypotheses, and doubts. Nevertheless, his writings enable us to conclude two things with certainty: he believed that reason-mediated experimentation was the only reliable source of knowledge, and he engaged in an intense and interesting experimental activity.

Keywords: scientific experiment; acoustics; Renaissance music; seconda pratica; vibrating strings; organ pipes

1. Introduction

Apart from being Galileo's father, Vincenzo Galilei (c. 1520–1591) was known in his day as a virtuoso lute player and a member of the Camerata de' Bardi, a group of musicians and scholars who sought to restore music to the splendor of ancient Greece. Today, Galilei is better known as a prominent music theorist. He was among the first to empirically investigate the monochord, treating it as a sonorous body, and formulate physical laws about it; for this reason he is often addressed as an early father of acoustics. He contributed greatly to the change in Renaissance musical style by proposing the restoration of Greek music and a diffuse use of dissonances. This eventually will lead to the so-called *seconda pratica* with Claudio Monteverdi (1567–1643). The brilliant lutenist Galilei became a theorist thanks to Giovanni Bardi, who sponsored his studies with the most important theorist of the time, Gioseffo Zarlino (1517–1590), in Venice for an extended period from 1561 to 1563. During this time, Galilei became interested in the history of ancient music, even more so than his teacher. Although less famous than his son, Vincenzo Galilei has also been extensively studied by science and music historians, even in recent times [1]. In particular, the role that experiments played in his studies has been the subject of careful analysis and received a wide range of assessments. One of the first works, from the 1970s, is by Stillman Drake, who sees Galilei as the true father of modern science being music the first experimental science [2]. This was followed shortly by a scathing article by Daniel Pickering Walker, who argues that Galilei was not a true experimenter and probably did not conduct any experiments [3]. H. Floris Cohen holds a somewhat more neutral position, arguing that some form of experimentation was present in Galilei [4]; Claude Palisca's opinion

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is closer to that of Drake [5]. An article by Adam Fix supports Galilei's experimental activity but claims that it was not experimentation in the modern sense [6]. Rather, it was a form of knowledge framed within what Fix calls artisanal epistemology. This is more of a sociological category than a philosophical one, though it has some followers [7]. In a recent article, Maurice Finocchiaro reaffirms the validity of recognizing Vincenzo Galilei as an experimenter, albeit not as skilled as Drake suggested. Finocchiaro refers to Vincenzo Galilei not as the father of modern science but as its grandfather, the predecessor to the father, suggesting a cultural link between father and son [8]. Eventually Galilei's experimental attitude was discussed by Athanase Papadopoulos in [9], where it was compared with that of Christiaan Huygens.

In this paper, building on previous studies, we attempt to clarify the role played by experiments in Galilei's approach to music by examining all of his works. Most of them has been published and translated into English. There are no known letters from Galilei on music; only those from Girolamo Mei to Galilei are available and easily accessible [10]. We hope to reach a reliable, if not definitive, conclusion.

2. Empiricist Instances in Galilei. The *Dialogo della Musica Antica et della Moderna*

Study of ancient music and interaction with Girolamo Mei led Galilei to a conception of music very different from that of his teacher, Gioseffo Zarlino. Reading the classics, especially Aristoxenus's and Ptolemy's works, made it clear to Galilei that there were other approaches to music besides those inspired by Plato and Pythagoras, which were in vogue during the Renaissance. This led him to break with authority and take an empirical approach. To overcome interpretive difficulties when reading ancient texts, Galilei consulted Girolamo Mei, a Florentine humanist. His interaction with Mei resulted in a musical approach that distanced him from Zarlino.

His first printed work, *Fronimo* (first edition 1569, second edition 1584) [11], a text on lute tablature, marked Galilei's departure from his former teacher. In his work, Galilei argued for the superiority of monodic music over polyphonic music. Here the string used to measure intervals was no longer a geometric line characterized only by length, as it was for most music theorists. Instead, it became a physical string of a given material and subject to a given tension (force of traction, due to weights or the rotation of tuning pegs), which influenced the pitch and color of its sounds.

In 1581 Galilei published the *Dialogo di Vincentio Galilei nobile fiorentino della musica antica et della moderna* (hereinafter *Dialogo*) [12], which attacked the ideas of Zarlino and the more traditional musicians. His writing provoked a harsh reaction from Zarlino which opened a heated debate that lasted until Galilei's death [13].

The *Dialogo* is today considered as a fundamental treatise of musical theory, and has been the subject of numerous studies and translations into English; such as [14,15]. The first 50 pages are dedicated to refuting Zarlino's thesis on Ptolemy's diatonic syntonic tuning. Pages 51 to 80 present the differences between the modal and tonal systems demonstrating the superiority of the ancient system. The ten best-known pages (80 to 90), critique modern madrigal polyphony. A section from page 90 to 138 focuses on ancient notation and instrumental music, here Galilei reported interesting historical documents, such as the hymns attributed to Mesomedes of Crete and Alypius's tables on Greek notation [12] (pp. 92–94). The rest of the *Dialogo* up to the end (pages 138 to 149), attacks modern instrumental music and presents the ancient music as a valid model [14] (Introduction).

In his treatise Galilei revealed his rejection of authority and his preference for empirical approach, calling for an agreement between sense and reason.

> Judicious and learned people are not content as the inexpert multitude is with the simple pleasure of viewing colors and diverse forms of objects, but with investigating afterwards the agreement and proportion of its elements and its properties and nature [12] (p. 86. Translation into English in [6]).

> I wish in those things which sensation can reach that authority always be set aside (as Aristotle says in the Eighth Book of the Physics), and with it the tainted reason that contradicts any [sense] perception at all of truth. For it seems to me that those who for the sake of proving some conclusion of theirs want us to believe them purely on the basis of authority without adducing any further arguments are doing something ridiculous, not to say (with the Philosopher) acting like silly fools [12] (p. 2. Translation into English in [5]).

Rejecting authority was not typical of Galilei; it was a general sentiment of the Renaissance. The same applies to the agreement between reason and sense. Although the spirit of the times played a role, Galilei had personal reasons that led him toward empiricism. He was familiar with the writings of Aristoxenus and Ptolemy, who carried on empirical instances in music, although it is unclear how much he understood of them given that he did not know Greek (Galilei could have had the opportunity to read Aristoxenus *Elementa harmonica* and Ptolemy *Harmonica* in the library in Venice during his apprenticeship with Zarlino. Moreover, he discussed them with Girolamo Mei in a lengthy correspondence that started in 1572 [10]).

According to Aristoxenus the understanding of music depends on hearing and reason, and hearing plays a major role.

> The geometer makes no use of the faculty of perception: he does not train his eyesight to assess the straight or the circular or anything else of that kind either well or badly: it is rather the carpenter, the wood-turner, and some of the other crafts that concern themselves with this. But for the student of music accuracy of perception stands just about first in order of importance, since if he perceives badly it is impossible for him to give a good account of the things which he does not perceive at all [16] (pp. 150–151).

Ptolemy had a more sophisticated epistemology, very close to the modern one. He was convinced that reason and hearing have distinct roles: “Hearing is the criterion for matter and condition, while reason is the criterion for form and causes” [17] (p. 68). Ptolemy specified that “hearing discovers what is approximate and accepts what is exact”, whereas “reason accepts what is approximate and discovers what is exact”. In other words, given two or more sounds, hearing discovers the approximate ratio of their pitches, determining whether they appear concordant. Then, reason examines the ratios and corrects them if they appear *irrational*, that is, contrasting with the hypothesis of harmonics. These rationally modified ratios and mathematical inferences are then subject to hearing, which could or should give its assent.

There are many salient points in the *Dialogo*. Probably the most important of them is the change in the epistemological status assigned to the relationship between music and numbers. Galilei distanced himself from the Pythagorean vision of his teacher Zarlino, abandoning the idea that numbers define the essence of music outside of any sensory reference, thus numbers are *sounding numbers*. According to Galilei, numbers were instead simply tools used to measure certain characteristics of sounding bodies particularly of strings, when they produce consonance or dissonances to the musician’s ear. This concept is expressed more clearly in later works.

> In this sense we can tell, by way of example, the ratio of the diapason through lines; and through them, as if born of them, the diapason may be said to lie between the half and the whole, or, if we wish, the one and the two. However, these must not be considered simply as cardinal numbers [numeri numerati], but as numbers measuring only those portions of the strings capable, when struck, of producing a pitch. For what Zarlino says in chapter of the first [book] of the *Istitutioni*, maintaining that number is sonorous, cannot stand in any case, for, not having in itself body—and sound is not produced without the percussion of some body capable of rendering a sound—a simple number, consequently, cannot be sonorous [18] (f. 45v. Translation into English in [19]).

Another important point of the *Dialogo* is Galilei's criticism of the contemporary polyphony that transformed the 'true' ancient Greek music into a 'false' modern music.

> Alongside the former, we can also include the Musici de Vulgari, who have a true understanding of practical music. They form and fill major and minor intervals using a variety of strings and accommodate them in accordance with the principles of counterpoint. As a result, when moving successively from one consonance to another, the ear, as I said of the eye, can desire nothing more. However, they are not familiar with the art of making music in a way that calms the intellect by expressing concepts through sound and voice, or inducing affection in others. Many of them firmly believe that they have found, known and perfected this art, but experience shows us that this is not true. Many of them would be able to put this belief into practice if they knew it, and they should be excused since until now they have had no insight into this matter, only much darkness and confusion of errors. They will not be able to excuse themselves in the future since it is clearly demonstrated to them that the purpose of true music is not what they have believed until now, but rather the purpose of false music [20] (p. 105).

Then there is the more technical criticism of the system of just intonation, which had become established in the Renaissance and had received Zarlino's approval with the idea of the senario. Zarlino had attributed the system of just intonation to Ptolemy, who had proposed the same tones and semitones of the just intonation in his *Harmonica* under the name of *diatonic syntonic genus* (It should be noted that the attribution of the just intonation to Ptolemy is not historically justified, for two reasons. First, just intonation had already been independently developed for musical purposes before Ptolemy's *Harmonica* was rediscovered by humanists. Second, just intonation did not play a privileged role in Ptolemy's work, but was presented alongside other diatonic genera).

Galilei argued against just intonation because the presence of two different major tones (9:8 and 10:9) posed many challenges in musical practice, and for this reason the system was not used by practitioners.

> I find through long observation that natural voices and instruments made by art do not actually play or sing in this modern music practice any of the nine ancient diatonic species in their simplicity. Our practicing contemporaries inadvertently use only three of them today, mixed together in different ways. These are the intense [diatonic] of Aristoxenus, the ancient ditonic diatonic, and the diatonic syntonic of Ptolemy [authors' note: Aristoxenus's intense diatonic genus divides the tetrachord into five equal semitones, the ancient ditonic diatonic genus (i.e., the Pythagorean one) divides the tetrachord into two equal tones and a limma, the diatonic syntonic of Ptolemy correspond to the just intonation [21] (p. 22)]. Among stringed instruments, I consider that the viola d'arco, the lute, and the

> stringed fretted lyre play the intense diatonic of Aristoxenus. Hearing and seeing in these instruments the uniformity of tones equally divided into two identical semitones impels me to believe that this is true, for the aforementioned intense diatonic of Aristoxenus was distributed in such a way, as you will understand in due course. However, the organ, the gravicymbalo, and the modern harp [. . .] in the division of tones, for example, the tones are separated into two unequal semitones. The wind instruments like direct flutes, transverse flutes, cornets, and other similar ones have power, due to the distribution of their holes and aided by the fine facility of their wise, expert players, to adjust to one [species) or another according to the need and their [player's] wishes. This is equally true of voices, but only when they are not willing to go against their nature and adapt to those [species]. Moreover, concerning the composing and singing of today, I am persuaded, because of what I told you and am about to tell you, that the ditonic diatonic is mixed with the syntonic of Ptolemy [12] (p. 20. Translation into English in [15]).

Galilei believed, in particular, that the system preferred for the lute, a fretted instrument, was Aristoxenus's intense diatonic genus. This genus had two equal tones and a semitone equal to half a tone. This essentially corresponds to what we now refer to as equal temperament.(Aristoxenus was associated with equal temperament because he divided the interval of a fourth into five equal semitones. However, this association was inaccurate because, like Ptolemy, Aristoxenus presented different ways of dividing the interval of a fourth without explaining his reasoning or preference). In the *Dialogo*, Galilei suggested a practical tuning method for this system: divide the lute string into equal semitone intervals (18:17). Twelve of these intervals cover the octave [12] (p. 49). However, the diatonic syntonic tuning system is preferable for the harpsichord. According to Galilei, this is because of the two instruments' different physical constitutions, particularly their strings. The lute's strings are made of gut, while the harpsichord's are made of metal. In the lute, the decrease in fifths in the diatonic scale produces a sweet sound. This is not the case for the harpsichord [12] (p. 48) Galilei claimed that the system of just intonation was not suitable for voices either. They mixe the diatonic and syntonic genera to avoid pitch migration, a phenomenon his contemporary, Giovanni Battista Benedetti (1530Ð1590), knew well [22].

Criticism of Zarlino's numerology is coupled with a reevaluation of Aristoxenus. His writings had been in circulation for some time, yet he was largely overlooked and regarded as a minor author (Galilei began translating Gogava's Latin edition of Aristoxenus's *Elementa harmonica* into Italian, but then left it to a more competent person [23] (p. 193); this translation survives in manuscript form in the Florence National Library [24]).

The Role of Girolamo Mei

Girolamo Mei (1519-1594), a Florentine humanist who had moved to Rome, has been vaguely familiar to historians of music as the author of a brief essay, the *Discorso sopra la musica antica e moderna*, published posthumously by Pier del Nero in 1602 [10] (p. 1). It was only in the 1950s that Mei was recognized in his true value as a musicologist, who played a fundamental role in Galilei's understanding ancient music and who was a source of inspiration for the *Dialogo*. A recent study that conducted a thorough textual comparison between Mei's published letters and his manuscripts has revealed that the *Dialogo* is heavily dependent on knowledge that Galilei never disclosed. In light of this, one could raise the serious question of the work's authorship, leading to a reevaluation of Girolamo Mei's musicological contributions. This dependence is evident not only from the textual comparison, but has also emerged through a detailed analysis of all the sources of the *Dialogo*, revealing that many of the numerous sources cited were not directly available to Galilei, who had to rely on cultural mediators, first and foremost Mei, in order to access them [25] (pp. 3–4).

3. *Discorso Intorno all'Opere di Messer Gioseffo Zarlino da Chioggia* and Scientific Papers

In 1589, Galilei wrote the *Discorso di Vincentio Galilei nobile fiorentino intorno all'opere di messer Gioseffo Zarlino da Chioggia* (hereinafter *Discorso*) [26] in response to Zarlino's 1588 work, *Sopplimenti musicali* [27], where the master fiercely attacked the pupil. The *Discorso* is a largely polemical text, essentially a refutation of the criticisms of Zarlino's *Sopplimenti*. However, the empiricist argument is more evident than in the *Dialogo*, and, for the first time in a text written by a music theorist, the Pythagorean thesis of the uniqueness of the ratios expressing consonances, (2:1) for the octave (diapason), (3:2) for the fifth (diapente) and (4:3) for the fourth (diatessaron), is clearly confuted.

> I wish to draw attention to two false opinions born among men, persuaded by the writings of some. I, too, had been among those. About these opinions, inasmuch as I am finally assured by means of experience, the master of things, I say this. They believe that the weights that Pythagoras attached to the strings in order to better hear the consonances were the same as those of the hammers from which he first heard them. Now, that this neither was nor could in some fashion be, experience (as I have said) demonstrates to us because one who wished to hear the diapason from two strings of equal length, thickness, and purity would of necessary suspend weights that were not in duple proportion (as were the hammers) but in quadruple. The diapente will be heard whenever weights of duple sesquiquarta [9:4] proportion are suspended from the same strings [. . .]. It is not true, therefore (and this is the other abuse), that the consonances cannot be had from other genera of proportions than the multiple and the superparticular. [. . .]
> Equally, we will have the diapason from pipes whenever the length and the void—or we might wish to say the diameter—of the low pipe is double the higher pipe. We will have the diapente from those where the diameter and the length are sesquialtera; and the diatessaron from those where their diameter and length be sesquitertia [26] (pp. 104–105. Translation into English in [13]).

To conclude with a general rule:

> So, the void of these pipes corresponds to the cube; the weights suspended from the strings correspond to the surfaces; and the strings simply stretched on the instrument correspond to the line [26] (p. 105. Translation into English in [13]).

Nothing is added to explain the reasons that led Galilei to his revolutionary statements; in fact, he seems to behave like one of the authorities he fiercely criticized, who must be believed because he is a competent person.

These statements will be taken up later in other Galilei's writings. This will be discussed below.

3.1. *The Role of Experience*

Today, the term *experiment* is usually accompanied by the qualifier *scientific*, i.e., scientific experiment. This locution refers to the observation of a natural phenomenon or its artificial reproduction, in order to verify a hypothesis. Experiments are assumed to be described in a way that allows them to be reproduced, which is essential for establishing truth. Moreover, the description is the work of an operator. In what follows, we will use the term experiment in a broader sense, essentially referring to the critical observation of natural or artificial phenomena relating to the physical world.

In Galilei's time, experience acquired through experimentation, which we will refer to as *critical experience*, was opposed to what we would call *ordinary experience*, a form of knowledge cultivated by philosophers that concerned the description of phenomena as they usually occur (i.e., as they mostly occur) or information reported in authoritative texts. The truth of information was not discussed. These two forms of experience differed in two fundamental aspects.

1. Ordinary experience was essentially static in nature, resulting from relatively simple items of information layered throughout the history of the members of a given society. In contrast, critical experience, especially at the beginning of the early science, was dynamic in nature, meaning it was constantly evolving.
2. Ordinary experience was based on authority and might also contain untrue statements, reported in written texts containing extravagant information, such as the claim that people from certain regions have two heads. Critical experiences involves gathering the testimonies of observers interested in reporting the facts as they have observed them.

In the Renaissance, critical experience was carried out not only by educated people, but also and primarily by craftsmen, such as engineers, painters, sculptors, musicians and alchemists. There was not much scope for experimental activity among educated people, as the various physical theories were too rudimentary to be tested. The most promising fields were those of mathematical physics (mixed mathematics). Of these, astronomy was closest to modern experimental activity and empirical observation began to be used to confirm or refute established theories, such as the immutability of the heavens and the immobility of the Earth. Mechanics owed a mature and very simple theory based on the laws of lever and inclined plane with regard to statics and machines. No one considered experimenting with laws that seemed logically necessary. The situation in dynamics was more interesting. The development of artillery had become important, yet the theories of motion in natural philosophy offered no assistance. Mathematicians therefore began to address the issue and brought the study of motion back into the realm of mathematics (this is one of Galileo Galilei's two new sciences dealt with in the *Discorsi e dimostrazioni matematiche sopra due nuove scienze* [28]). Apart from the need for theoretical clarification, experimentation was difficult due to the lack of precise instruments, particularly for measuring time. In essence, it can be said that scholars' recommendations did not provide operators with specific information and were not particularly useful in solving the problems that arose in the construction of new buildings and machines, for metallurgy and other activities.

Music was in a unique position. Its theory was highly developed, even though the nature of sound remained unknown; we would have to wait until Giovanni Battista Benedetti came along to understand that. The theory was developed by mathematicians and music theorists alike, with Zarlino being a notable figure among them. Many practitioners had a basic understanding of music theory and could verify theoretical ideas because they had a very sensitive measuring instrument: their ears.

Galilei's Experiments

Vincenzo Galilei used in his writings the words *sperienza* and *esperienza* with different shades. Nevertheless, it seems to us that in any case his was not the ordinary experience, but the critical one. It should be stressed that Galilei's account of his musical experiments does not meet modern standards. It does not include numerical evaluations in the form of tables, for example, and never provides precise information on how the experiments were carried out. The only certainty is that he relied on his keen sense of hearing. At most, Galilei asserted that he had conducted the experiments many times, which makes them very difficult, if not impossible, to replicate.

Experiments are referred to through the *Discorso*. However, they are mostly present in two short papers, *Discorso particolare intorno alle forme del diapason* [18] and *Discorso particolare intorno all'unisono* [29], named *scientific papers* after Palisca who transcribed and translated them from Galilei's unpublished manuscripts [19].

Very interesting examples of the refute of the authority and of experimental attitude is given by the following experiences:

> I am now coming to say with greater strength, that the fifth in the 3:2 proportion is the most perfect, sweeter than it is in any other ratio, as I have judged by ear after many many experiences [*sperienze*] (for I know no better means for attaining certitude) [26] (p. 117. Translation into English in [13]).

Here, whereby, at his time, was assumed as an axiom of music—that is that the fifth is the sweeter consonant after the octave—is considered worthy of experimentation. Actually, in the *Dialogo*, Galilei had contradicted the axiom, at least when referring to the lute and Aristoxenus' tuning, such as that of the lute:

> The fifth, within the sesquialtera, not only seems to possess, but actually does possess a small degree of hardness due to the extreme amount it can be augmented, which avoids my saying (together with others of delicate hearing) that it is harsh. In the manner of Aristoxenus, however, it seems that the small degree of diminution gives it grace and causes it to become more in keeping with the taste of today, that is, soft and languid, and I do not believe that that happens for any other reason except being accustomed to hearing them continually under that form, or a similar one [12] (p. 55. Translation into English in [15]).

> For among coins of the same goodness of material, of the same weight, cavity, thickness, and height, and also among little copper bells poured from the same mold, I have often found a difference of a whole tone [18] (f. 47r. Translation into English in [19]).

The first experience, according to Palisca, would not be considered as an experiment but an attentional observation, and the judgment would not be about a fact but about an "aesthetic observation"; in the second experience, however, it would be repeated trials with the same material and procedure; it would then be a contrived experiment [5] (p. 144).

In line with what we have said about acquiring experimental facts, we do not believe there is any significant epistemological difference between the two experiences. Of course, if we are referring to modern experimentation, there is a difference. In the second case, the experiment could be carried out in a laboratory using equipment capable of measuring frequencies, thus eliminating the need to rely on the observer's sensory abilities. In this instance, it would be considered a classical acoustic experiment. In the first experience, instead, the observer cannot be avoided. In any case, the assertion that we are not dealing with a fact, but rather with an observation related to personal preference, does not stand up to scrutiny in light of modern epistemology.

At the end of the *Discorso particolare intorno alle forme del diapason*, Galilei discussed his experiments on the variation in sound perception when pitches vary continuously, as can be done by moving a finger along the neck of a fretless viola. Then he referred to the sound produced by a glass vase when rubbing its rim with a finger and varying the water level.

> We cannot have any means of modulating from a low to a high pitch and from high to low through continuous quantity except by tightening or relaxing a string without stopping it with the finger; or from a glass vase by raising or decreasing the level of water while revolving the finger on the surface of the rim of this vase or glass, however you want to call it. In these modes the ear also, because of its imperfection, judges the effect in the same way that we said it judges the stopping of a string on the fingerboard of a viola [18] (f. 54v. Translation into English in [19]).

This same phenomenon was reported by Galilei's son in his *Discorsi e dimostrazioni matematiche sopra due nuove scienze* [28] (p. 142).

The experimental approach is particularly evident in his *Discorso particolare intorno all'unisono* [29], which illustrates Galilei's critical spirit and the fundamental role he entrusted to experience. The subject of this paper was unison, and it was not so much a discussion of whether it is a consonance or not, but rather of the different ways in which unison is realized, with the understanding that it can never be perfect. Galilei considered unison from two different definitions:

1. Direct and empirical, according to which, for example, two strings are in unison when they produce two sounds of the same pitch.
2. Indirect and mechanical, according to which two strings are in unison when they are of the same material, length, diameter, and tension [21] (p. 104).

Galilei believed that in neither case could perfect unison be achieved in practice. Even if two strings were perfectly in unison (definition 1), but made of different materials or cross-sections, they would produce sounds of different colors, and thus it can be said that they are both in unison and not in unison:

> It is possible to stretch to a unison two strings of the same material, length, and goodness, but if the thickness is different, their sound will depart from a true unison to the extent that there is an inequality. When the sound of these strings reaches the hearing-at least that of persons versed in such matters-the hearing will recognize an audible difference [29] (f. 56v. Translation into English in [19]).

> Now I shall ask those [who believe it is possible to tune in unison] whether they detect any difference in their sounds when playing a string of copper or steel and then or at the same time one of gut of the same thickness, length, and goodness in unison with the first and fulfilling whatever other conditions according to their method. If they answer truthfully, they will say yes [29] (f. 57r. Translation into English in [19]).

> They reply that, given any two strings stretched to a particular tension, I can raise or lower the pitch of one more or less than the other. I answer yes, and they reply that, since I can raise or lower one more than the other as I wish, I can make them equal and unisonant. In this case I reply as Aristotle did to those who denied the squaring of the circle, for they responded just as do the people I am talking about. Aristotle says, then, that since you can find a circle greater than a [particular] square and another smaller than that square, you can consequently find one that is precisely the size of the [required] square. I do not deny that this can happen, but I say as well that this circle equal to a square has been sought by the greatest intellect this world has known as long as the world has existed, yet it has not been found, so far as I know [29] (f. 59r. Translation into English in [19]).

Galilei suggested an interesting experiment to demonstrate how it is essentially impossible to tune two strings, one of steel and the other of gut, in unison. While it is true that the ear perceives the strings as being in perfect unison when they are plucked in their entirety, when they are divided into short portions with frets, as in the lute, the minute differences that were previously imperceptible are now accentuated.

> If I place on a lute one gut string and one steel string and stretch them to be in unison in their way; then if, for example, I position seven frets and I pluck the open strings, or if I position twelve frets (and then pluck the open strings), they will not be in unison [. . .]. The same happens to the ear as happens to the sight when two lines are so close to being parallel that in the space of a hundred paces they depart only by a fourth of an arm's length. This difference divided by one hundred is so minimal that it would not be detectable by the sense, nor would the sense discern it in the space of a single pace, but in one hundred, yes [29] (ff. 59r–59v. Translation into English in [19]).

Finally, Galilei used some musical examples to demonstrate how ideas derived from theory could lead to erroneous results when applied uncritically. He pondered whether these examples would be more enjoyable if performed by a skilled singer or musician playing an instrument like the lute, as it would appear to be the case.

> I now ask them whether a song sung with discretion by men of judgment, acute hearing, and good vocal disposition would not be better than when played by some excellent player on an instrument tempered according to the intense tuning of Aristoxenus [29] (f. 61r. Translation into English in [19]).

He then concluded:

> They will reply after hearing it sung and played that it satisfies them more played than sung. And they will speak correctly for reasons I will give now. Voices, being naturally perfect, when well developed4 by the art of singing, cannot sing well a song that is not composed according to their perfect usage, but an instrument tempered according to the imperfect usage in which this song is imperfectly composed, on the other hand, can play it [29] (f. 61v. Translation into English in [19]).

3.2. Experiments on Vibrating Strings

In the *Discorso particolare intorno alle forme del diapason*, in some ways, Galilei reiterated what he had previously written in the *Discorso*, but he also introduced an alternative way of expressing the consonances of the strings, derived from their cross-sections..

> The second way of hearing the same diapason, which I said by way of example was through surfaces, is between two strings made of the same material, equal in length, thickness, and goodness from which weights are hung related to each other not as the duple [ratio], as some have said, but in quadruple proportion. From the sound of these strings when they are struck at the same time a diapason will be heard [18] (f. 46v. Translation into English in [19]).

> The same thing will happen if equal weights are suspended from two strings the thicknesses of which are in quadruple proportion, provided the length and goodness are the same [18] (f. 46v. Translation into English in [19]).

The modern reader struggles with the fact that there is still no description of how these highly relevant results were obtained. Only a few lines later, we are told that Galilei obtained some results through repeated experimentation.

> I believe that it [the diapason] may be obtained through other terms and through other means, as by striking rods, vases, copper and silver coins, and other things. But these will be vague and not the determinate weight, size, and number demonstrable with strings and pipes. For among coins of the same goodness of material, of the same weight, cavity, thickness, and height, and also among little copper bells poured from the same mold, *I have often found a difference of a whole tone* [emphasis added] [29] (f. 47r. Translation into English in [19]).
>
> This same variety of the ratios of the diapason may be found also among strings of the same material, equality of length, and goodness, but of uneven thickness, when the same quantity of weight is suspended, and in other ways *that I have experimented with many times* [emphasis added] [29] (f. 47r. Translation into English in [19]).

Notice that the above quotation could be seen as the only claim Galilei made on direct experiment on strings of different cross-section. There are no such claims for what tension is concerned.

What is certain is that Galilei's results are correct and are reported in written form for the first time. Therefore, it seems inevitable to conclude that he obtained them through more or less refined experiments. Although the idea that the pitch emitted by vibrating strings increases with tension and decreases with string cross-section was well known to practical musicians and many theorists—as reported, for example, by Ptolemy [17] (p. 27)—no one dared propose quantitative laws.

The second part of Galilei's law, which states that the pitch decreases by an octave as the cross-section increases four times, is likely the easiest to justify. Galilei used a lute with six or eight courses of strings separated by intervals of a fourth, except for the central strings, which were separated by an interval of a third. Since the tension of the strings was more or less constant and their length and material were the same, the pitch differences must have been due to differences in size. A modern reader would certainly refer to the diameter, but it was difficult to measure this quantity with precision in Galilei's time. For example, a chanterelle (the thinner string) measured about 0.4 mm, and the closer string measured a little more than 0.5 mm (see Appendix A). There were no instruments that could measure the diameter with sufficient precision to differentiate between them (even a tenth of a millimeter would be considered a high degree of precision). It was easier to measure the cross-section; this could be done indirectly by weighing long strings of known specific weight, which was generally that of gut. However, notice that if Galilei had considered the diameter, he could not have said that the ratio was (4:1) because the corresponding ratio for diameters, as for lengths, was (2:1).

Obtaining the first part of the law, that of the quadruple of the tension, is more difficult. First, Galilei could not directly measure the tension of the lute strings because he could not measure the pegs' rotation. Additionally, he had to be careful not to break the strings. For instance, raising a chanterelle, the thinner string of the lute with a diameter of around 0.4 mm, by an octave could cause it to break. However, it would be possible to increase the tension fourfold for an intermediate string with a cross-section of at least four times, such for example that of a bass string with a diameter of around 1 mm. This string could be raised by an octave without breaking because, even if the force were increased by a factor of four, the stress would still be less than the operating stress of the chanterelle.

The only way Galilei could control the tension for the string was to use a monochord, equipped with two strings, and tuning them at the unison. The weight pulling on one of the strings was then increased until a higher octave was obtained. This should occur with a weight four times greater than that of the reference string. In reality, there is no need to reach the higher octave, reaching the fifth is sufficient, which requires a weight slightly more than twice that of the reference string.

Even assuming that the experiment was carried out using this procedure, it is unlikely that Galilei could have obtained the result he claimed; what he could probably have obtained by repeating the experiment several times with different types of strings, were ratios close to four to one. However Galilei was not a modern experimenter; he was conditioned by the mathematical instruments of the time and also wanted to arrive at a simple, round result.

Although experimentation seems to be the only way to explain Galilei's results, he could have done so in a much less sophisticated way than described above. Perhaps he did so simply by tuning the strings of his lute. Maybe he did not have the proper string available and saw that, with the right amount of tension, he could tune a string intended for F_3 to B_3, a fourth above. He may have even been able to reach two fourths or an octave to an F_4. Even if he did not know the exact tension, it may have been clear to him that it was greater than twice as much. With a little imagination, he could have said it was four times greater.

Mersenne Law

The results referred to in the *Discorso particolare intorno alle forme del diapason* can be summarized as follows:

1. The pitch of a string made of a given material, with a given cross-section and tension, goes down by an octave when its length is doubled.
2. The pitch of a string made of a given material, with a given cross-section and length, rises by an octave when its tensile stress is quadrupled.
3. The pitch of a string made of a given material, with a given length and tensile stress, goes down by an octave when its section is quadrupled.

A modern reader is tempted see in the previous items what is known as Mersenne's law [30] (Second partie. Livre troisiesme des instruments á chordes, pp. 123–125). Actually, Galilei could not have seen this for two reasons. First, he had no concept of the frequency of string vibrations or the ability to associate pitches with frequencies and an octave with the frequency ratio of (2:1). Second, he was not interested in the acoustics of vibrating strings; he only wanted to show that the octave could be reached with different frequency ratios to contrast Zarlino's senario. However, Galileo Galilei, son of Vincenzo, was able to formulate the law of vibrating strings, and he refined the third point by replacing the section with the weight per unit of length:

3. The pitch of a string with a given length and tensile stress, goes down by an octave when its weight for unity length is quadrupled.

> Here note that the heaviness of the moveable is more resistant to speed than is its thickness, contrary to what one might at first suppose, since it seems reasonable that speed should be more retarded by the resistance of the medium to being separated by a thick but light moveable than by a heavy and thin one; yet in this instance, the contrary happens [28] (p. 146. Translation into English in [31]).

This makes the law more general, something that even Mersenne did not fully understand.(Galileo published his results in the *Discorsi e dimostrazioni matematiche sopra due nuove scienze* of 1638 [28] (pp. 141–150), two years later than Mersenne, however it is much probable that his result was much anterior). Though it seems plausible that the soon took from the father, no textual evidence at the momant has been found.

3.3. Behavior of Organ Pipes

Some historians have taken the statement on pipes as evidence that Galilei did not conduct any experimental tests on pipes or perhaps even on strings. They believe he allowed himself to be carried away by questions of aesthetics or numerology to obtain results that would enable him to argue with Zarlino. For example, Walker wrote:

> It is evident that Galilei did not do any experiments, since the pitch of a pipe is a function of its length and not of its cubic capacity. In his unpublished *Discorso intorno alla diversità delle forme del diapason*, he asks the question: what interval would be given by two pipes of the same diameter but one of which is double the length of the other? and answers that it could be an equally tempered major third [32] (p. 24).

The thesis is supported by the fact that Galilei devoted less space to the description of experiments than to (perhaps) numerological considerations.

For example, Galilei argued that the ratio (8:1) was the true expression of the octave, because all consonances, including fifths and fourths, are contained in the sequence 1, 2, 3, 4, 5, 6, 7, 8, in the sense that, for example, the fifth, characterized by the ratio (3:2), and the fourth by (4:3) are obtained with numbers belonging to the sequence, while in the case of the octave measured by the ratio (2:1), the fourth and the fifth are expressed by the numbers 3, 4, which are outside the sequence. In addition, the sequence from 1 to 8 also contains the number 7, which for Galilei is very important in music because it allows to obtain the replicas of the different intervals with a simple addition (for example, adding 7 to the fifth gives the twelfth, that is, the replica of the fifth) [21] (p. 103).

These observations could be seen as either an acceptance of the esoteric power of numbers and a return to Zarlino's numerology, which was widely despised, or as a useful coincidence. In both cases, Galilei's assertion that all consonances can be found between the numbers one and eight is open to criticism. For example, the ratio (8:5) does not express a minor sixth for pipes. When related to the string, it corresponds to the interval $(2 : \sqrt[3]{5})$, which is not even an interval in the strict sense because it is irrational. It is difficult to determine the origin of the law of organ pipes because the law provided by Galilei is substantially incorrect.

There may be some truth in what Galilei's critics say about the pipes, but it should be pointed out that the behavior of pipes is not so easy to grasp, or at least not as easy as that of strings, even though organ pipes had been built for centuries according to well-defined criteria reported in musical manuals. If a modern reader had picked up a manual for organ builders (e.g., the 12th century manuscript *De commensuralitate fistularum et monocordi* by Gerbert d'Aurillac (c. 950–1003, Pope Sylvester II) is interesting [33,34]), he might have said, "But, look, it has long been known that organ pipes have a length ratio of to two to one for an octave, so what does Galilei's ratio of one to eight has to do with it?". The fact is that organ pipes did not all have the same cross-section, which was generally greater the longer they were, and moreover, the octaves did not rise exactly by a ratio of two to one, a reason that will become clear later: they rise by a ratio of two to one, not on the *geometrical lengths* but on the *acoustic lengths*, which may be much greater. So it was not so strange to associate the pitch of the pipe with its volume rather than its length. After all, Mersenne himself, a generation later, would take into consideration the same law as Galilei. In fact, for a mathematical expression showing that the fundamental frequency of an organ pipe is proportional to the inverse of its length, one has to wait for Leonhard Euler, more than a century after Galilei's paper [35,36]. Euler presented his theory in his *Dissertatio physica de sono* of 1727, where he relied on the analogy between the string and the column of air contained in the pipe. The length of the string is for the length of the pipe, the tension of the string is for the atmospheric pressure, and the specific weight of the string is for the specific weight of the air [35] (p. 9).

Thus, it should be ruled out that Galilei did not conduct experiments, as some claim. Even though for him, a musician who probably discussed the matter with lute players, experiments with strings were simpler than those with pipes.

If Galilei had only considered similar pipes, then the thesis that the octave is produced by a ratio of 8:1 would also be empirically justified. In fact, in similar pipes, halving the length to obtain the upper octave, the volume decreases by a factor of eight. Unfortunately, he meant the triple ratio in general, as is clear in cases where he compared the sounds of two pipes with the same section but twice the length of each other, where there is a double ratio of volume: "What sort of interval would two pipes of the same diameter but double the length produce? A major third of the intense tuning of Aristoxenus, which is actually the third part of the octave" [18] (f. 50v).

Galilei was referring here to Aristoxenus' intense diatonic tetrachord, in which there are two whole tones and one half tone in a fourth, which he identified with equal temperament. An octave is made up of six tones, and a third is made up of two tones, each of which is $\sqrt[6]{2}$ (modern notation), and two tones are $\sqrt[3]{2}$, which corresponds to the third part of the octave. Of course experimenting with halving the length of a pipe would have given more or less an octave above and not a third, and Walker criticism is justified. Palisca suggested the possibility that Galilei could have confused length with thickness (section) [19] (p. 191). Indeed, if one were to compare two pipes of equal length and one twice as thick as the other, because the acoustic length of the ticker pipe can be much greater than that of the thinner pipe, one could hear something similar to a (minor) third, as noticed by Mersenne [30] (Part 2, Book 6, prop. 12, pp. 331–332). This would justify Galilei observation.

4. Conclusions

A thorough analysis of Galilei's texts clearly reveals his empirical approach, evident in his reliance on observational studies and experiments (see Appendix B). However, his references to experiments that he said to have verified several times without further comment are not satisfactory to modern readers who are accustomed to precise results supported by numerical tables; another source of dissatisfaction for them is the complete absence of any description of how the experiment is carried out, which would allow it to be reproduced.

Galilei was certainly motivated by an underlying ideology, at least when he spoke of organ pipes. In the case of vibrating strings, however, he obtained significant results: linear laws for lengths and quadratic laws for tensions and cross-sections of strings. Therefore, it was natural for him to think that there might be another way of expressing consonances using cubes, thus completing the sequence 1, 2, 3. It is more or less certain that he did not conduct experiments with specially prepared pipe systems.

We believe that instead of focusing on Galilei's inaccuracies, one should consider why he came to undermine the established order of numerology by going so far as to challenge even the 'holy' Pythagoras. There is no evidence that other scholars—at least we are not aware of any—suggested the idea to Galilei, nor are there any theoretical considerations that could lead to this result.

Certainly, the reading of Aristoxenus, whose writings had recently been published, played an important role, with its focus on empirical aspects, along with the loss of importance that the principle of authority experienced in the Renaissance. But if these are the necessary conditions that made Galilei's idea possible, they do not fully explain it. The fact that Galilei was first and foremost a lutenist and not a theorist was certainly the most important stimulus. While tuning the lute, he was touching the change in pitch of a string as the tension varied, and it seems likely that he was driven by curiosity to see what would happen if he increased the tension beyond what was necessary for tuning, first to the fifth and then to the octave. When he saw that the Pythagorean order had been subverted for the strings, he probably doubted the result for the pipes as well [21] (p. 104).

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Appendix A. Lute Strings in the Renaissance

During the Renaissance, the lute typically had six courses of gut strings (Galilei also used eight-stringed lutes) covering approximately two octaves, separated by intervals of a fourth, with the exception of the two middle strings, which were separated by an interval of a third; using a modern notation the notes of the lute can be expressed as follows:

$$\mathrm{G_2\ C_3\ F_3\ A_3\ D_4\ G_4}$$

The course giving the highest note, $\mathrm{G_4}$, called *chanterelle*, had a frequency of vibration of about 400 Hz, a cross-section of about 0.4 mm, and was stretched close to its breaking point (about 300 MPa), with a tension of about 30 N. The other strings had variable diameters up to a maximum of about 1.5 mm, and maintained more or less the same tension. Apart from the chanterelle, all the string courses are doubled: the first two in unison and the other two in octaves.

In Galilei's time, the resistance of the strings was not measured as the tension per unit area (i.e., stress), but reference was made to the maximum frequency that could be played with that string. Similarly, today, the resistance of the string is measured by the product of the maximum frequency (f_{max}) and the string's length (l). We know that the maximum frequency emitted by a vibrating string is given by the following formula:

$$f_{max} = \frac{1}{2l}\sqrt{\frac{\sigma_Y}{\rho}}$$

where σ_Y is the breaking stress, and ρ is the specific mass. It is clear that for a given material, i.e., a given ρ value, σ_Y is related to $f_{max}l$. For Renaissance gut strings, we can assume, at least approximately, $\sigma_Y = 340$ MPa, and $\rho = 1300\ \mathrm{kg/m^3}$, from which we have $f_{max}l \approx 256\ \mathrm{m/s}$. Assuming the length of the string $l = 0.6$ m as typical for the lute, it is $f_{max} \approx 427$ Hz.

Appendix B. Table of Galilei's Main Musical Experiments

1	Modern music practice as a matter of fact does not play as the ancient diatonic species	[12], p. 20
2	Pipes correspond to the cube, weights to the square, and lengths to the line	[26], p. 20
3	Comparison of the various consonances with the just fifth	[26], p. 117
4	Rising the finger of the surface of the rim of a glass with different levels of water	[18], f. 54v
5	Observing the different colors of unisons from strings with different thicknesses	[29], f. 56v
6	Impossibility of a perfect unison	[29], ff. 59r–59v
7	Comparison of the same song sung by a good singer and an excellent player of an equally tempered instrument	[29], f. 61r
8	The octave can be obtained by striking rods, vases and coins	[18], f. 46v
9	In strings of the same length and tension the octave is obtained with thicknesses which are in quadruple proportion	[18], f. 46v

Legend: [12] *Dialogo*; [18] *Discorso particolare intorno alle forme del diapason*; [26] *Discorso*; [29] *Discorso particolare intorno all'unisono.*

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