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METHODOLOGIES AND APPLICATION

A binary differential evolution algorithm for airline revenue management: a case study

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Abstract
In the current highly competitive airline market, many companies have failed due to their low revenue rates. For this reason, many of them have to develop strategies to increase their revenue. In this study, we develop revenue management (RM) strategy for the Iranian airline industry. More specifically, we present a mathematical model that considers some conditions not studied in previous research in order to provide a more realistic RM modeling of airlines that fits well for the special characteristics of Iranian Airways. A binary differential evolution algorithm is employed to solve the model due to the stochastic nature of data and the NP-hardness of the considered problem. To generate maximum revenue among the six types of airplanes that fly the four capital cities of Iran, the airline under investigation is advised to operate only 21 flights to those cities and cancel the rest of the flights.

Keywords Revenue management · Airline industry · Optimization · Binary differential evolution · Booking · Overloading · Cancelation

1 Introduction

The airline industry, as a part of the transportation industry, plays a fundamental role in the economy of many countries. Due to the popularity of air transports, the number of users of this service is increasing dramatically. As a result, many flight agencies join the airline industry. Besides the increasing demand for air transportation, the economic performance of airlines has become one of the major concerns.

Revenue management (RM) is a promising and useful management technique that aids companies to maximize their revenues through mathematical approaches. The origin of RM is in the airline business. Later on it has been effectively applied to various other industries including hospitality, car rental services, attended home delivery and even manufacturing industries (Klein et al. 2019). Reviewing the related literature shows that many researchers have devoted their attention to RM applications in service industries (e.g., Wang and Brennan 2014; Guadix et al. 2011; Denizci Guillet and Mohammed 2015; Abrate and Viglia 2016; Heo 2017). A joint feature of the mentioned industries is that they offer diverse services at distinct prices before the factual service time; otherwise, they might not be able to benefit from well-timed selling of

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their products. In the airline industry context, seats on flights would be those services. Airline companies need to attract passengers with low costs and superior services. Many studies have considered various options to solve the mathematical models in order to optimize airline firms' revenues (e.g., Lin et al. 2017; Grauberger and Kimms 2016a, b; Aslani et al. 2014; Aydın et al. 2012; Zhang et al. 2010; Gosavi et al. 2007; Mukhopadhyay et al. 2007; Zhang and Cooper 2005).

Due to its geopolitical location, Iran is a highly competitive airline transportation hub in the Middle East. Approximately, fifteen airlines currently operate in Iran for the passengers' transportations from/to Iran and abroad. Nowadays, there are many private and public airline companies in Iran, which compete with each other. In the current study, we aim to focus on the biggest public airline company in Iran, denoted here as Company X, which is also the oldest airline in this country. Their planes are very old fashioned, and their likelihood of crashing is comparably high. That is why frequently people are afraid to travel with them. Consequently, this company is on the verge of bankruptcy. New emerging companies, on the other hand, are using newer planes for traveling, dramatically increasing their market share. Therefore, Company X needs effective planning and management for its planes to optimize its revenue.

In this study, we address the mentioned problem by implementing the RM in the Iran airline market for a two-year period during the years from 2015 to 2016. We propose a mathematical model for revenue optimization of the considered company using a variant of the differential evolution (DE) algorithm (Storn and Price 1997) for binary variables, the binary differential evolution (BDE) algorithm (Pampara et al. 2006). The primary contribution of this study is to provide different options and methods to solve the above problems. In previous studies, scholars have considered the availability of seats, overloading, price optimization, cancelation and booking using linear programming (Gosavi et al. 2007; Zhang and Cooper 2009; Oancea 2016), system dynamics (Sierag et al. 2015), and network programming (Grauberger and Kimms 2016a). In the current research, we consider three options for cancelation, booking, and overloading and their relevant penalties simultaneously, which have not been discussed in previous research, particularly in an Iran Airways context. We use stochastic data for these options to make our presented model more realistic. The stochastic data are in the form of intervals containing lower and upper bounds. Then, we model the investigated revenue optimization problem by formulating it as an integer-programming model.

The DE algorithm is a kind of optimization approach that adopts some concepts from nature, particularly the evolution of organisms. This kind of metaheuristics is very popular, because it can cope with complicated optimization problems, usually leads to fast and sufficiently good solutions, and can be easily applied by non-experts. In particular, in solving NP-hard problems, DE proves to be an efficient evolutionary algorithm with acceptable results (Karaboga and Cetinkaya 2004; Mayer et al. 2005). Therefore, we utilize the BDE algorithm to solve the integer-programming model due to the existence of binary variables. In addition, the BDE algorithm is an efficient stochastic optimization algorithm, the use of which makes sense to solve an NP-hard model with stochastic data (see Wang et al. 2012). It is noteworthy that the differential evolutionary algorithm has been previously applied to different problems and has been used for various applications (e.g., Varadarajan and Swarup 2008; Qin et al. 2009; Wang et al. 2012; Tang et al. 2014; Wang et al. 2017). Other applications include power systems (Chen et al. 2015; Qian et al. 2017), location management (Swayamsiddha et al. 2017), health care (Liang et al. 2017; Zhao et al. 2017; Baraldi et al. 2018), and satellite scheduling (Azizi and Sajedi 2017).

This paper includes seven sections. Section 1 is the introduction. Section 2 reviews the RM and its application to the airline industry. Section 3 is about the proposed model. The case study is described in Sect. 4. Section 5 focuses on the results of the paper. Section 6 describes our solution method. The final section presents the concluding remarks.

2 Related works

During the past three decades, many companies have realized the importance of revenue or yield management. The airline industry is one of the industries in which implementing RM has led to some advantages (Vardi et al. 2016). It is evident from the literature that there are many studies of revenue optimization in the airline industries (such as Parker 2003; Yu et al. 2004; Oliveira 2003; Graf and Kimms 2013; Wittman and Belobaba 2017; Grauberger and Kimms 2016a, b; Chao and Li 2017).

Reviewing previous studies demonstrates that researchers have proposed airline RM models considering several relevant options. The seat control is one of the options that has been extensively examined in earlier studies. Several researchers such as Williamson (1992), Bertsimas and De Boer (2005), Zhang and Cooper (2005) and Klophaus and Pölt (2007) have considered flight ticket booking as an option to the RM models. In addition to booking limits, flight cancelation is another important option that is found in the previous RM models. Some studies such as Gosavi et al. (2002), El-Haber and El-Taha (2004), Gosavi et al. (2007), Aydın et al. (2012), Sierag et al. (2015) and Vardi

et al. (2016) have simultaneously considered booking and cancelation limits in their RM models in the area of airline business.

Besides the mentioned seat control options, some studies have devoted their attention to competition options among airlines and the strategic alliances of airlines. In this regard, some game theory models like a noncooperative game model (Hu et al. 2013), a duopoly game model (Li et al. 2016) and Markov-game model (Wright et al. 2010) have been applied to find the equilibrium strategy for airlines under competition. Ticket pricing is also one of the most important factors used by airlines to compete in the market. Various researchers have focused on the pricing option like Zhang and Cooper (2009), Birbil et al. (2013), Li and Pang (2017), Aslani et al. (2014) and Otero and Akhavan-Tabatabaei (2015) to find the optimal pricing strategies. Furthermore, a few other options, which have been considered in the previous researches, include buyback policy (Lin et al. 2017) and commitment (Aydın et al. 2016). Nonetheless, these options are less used in the airline RM models.

Loading is also another factor that is seldom studied in airline RM research. Loading refers to the allowed weighting of cargo for each passenger on a specific flight. Recently, Chao and Li (2017) have investigated the optimal loading option and have studied the optimal loading density for a flight. However, they have studied air cargo companies instead of passenger airlines. Considering the fact that investigating the overloading option is less considered in the airline RM literature, in this study we consider three options of overbooking, cancelation, and overloading to optimize airline revenue. To the best of our knowledge, this is the first study that simultaneously considers the three mentioned options in the airline RM context. Table 1 highlights the mentioned research gap this study aims to fill, and Table 2 demonstrates methods of solving revenue management problems.

Focusing on the solution methods in airline RM models indicates that the most applied method for solving the presented models is simulation-based optimization models (e.g., Bertsimas and De Boer 2005; Zhang and Cooper 2005; Gosavi et al. 2007; Graf and Kimms 2013; Doreswamy et al. 2015). It should be noted that a metaheuristic approach which well matches the stochastic environment of RM has not been utilized until now. This motivated us to use a BDE algorithm to solve our presented model.

3 Proposed RM model

In this section, we introduce the airline revenue optimization model taking into account three decisions regarding booking, cancelation, and loading. The mathematical model is explained as follows:

3.1 Notations

I	Number of origins, also used for the set of origins
i	Index of origins $i = 1, 2, \ldots, I$
J	Number of destinations, also used for the set of destinations
j	Index of destinations $j = 1, 2, \ldots, J$
V	Number of planes, also used for the set of planes
v	Index of planes $v = 1, 2, \ldots, V$
B	Number of booked seats, also used for the set of booked seats
b	Index of seats booked by passengers $b = 1, 2, \ldots, B$
C	Number of canceled seats, also used for the set of canceled seats
c	Index of seats canceled by passengers $c = 1, 2, \ldots, C$
O	Number of passengers with overloading, also used for the set of passengers with overloading
o	Index of passengers with overloading $o = 1, 2, \ldots, O$

3.2 Parameters

P_{jvb}	Profit by booking
P_{jvc}	Profit by cancelation
P_{jvo}	Profit by overloading
$P_{x_{ij}}$	Price of flight from origin i to destination j
$N_{x_{ij}}$	Number of passengers in a plane from i to j
G_{ijvc}	Total expected profit from ticket cancelation
D_{ijvb}	Total expected profit from ticket booking
H_{ijvo}	Total expected profit from overloading
$\Pr(c)$	Probability of canceling ticket (between 0 and 0.2)
$\Pr(o)$	Probability of overloading (between 0 and 0.03)
$\Pr(b)$	Probability of ticket booking (between 0.8 and 1)

3.3 Decision variable

$$x_{ijv} = \begin{cases} 1 & \text{if plane } v \text{ goes from origin } i \text{ to destination } j \\ 0 & \text{if plane } v \text{ does not go from origin } i \text{ to destination } j \end{cases}$$

Suppose that there are V airplanes wi different capacities belonging to the set $V \in \{v_1, v_2, \ldots, v_V\}$. These planes go from the origins identified by i where $i \in \{i_1, i_2, \ldots, i_I\}$ to the destinations shown by j where $j \in \{j_1, j_2, \ldots, j_J\}$. Based on the kinds of planes and the number of destinations, we can determine the decision variables. The first

Table 1 Considered options of airline RM model in the recent literature

Author(s)	Considered options					
	Booking	Cancelation	Overloading	Pricing	Competition	Buyback policy
Gosavi et al. (2007)	√	√				
Zhang and Cooper (2009)				√		
Aydın et al. (2012)	√	√				
Birbil et al. (2013)	√			√		
Aslani et al. (2014)				√		
Grauberger and Kimms (2016a)	√				√	
Grauberger and Kimms (2016b)	√			√	√	
Oancea (2016)	√			√		
Li et al. (2016)	√				√	
Chao and Li (2017)			√	√		
Lin et al. (2017)	√					√
Yoon et al. (2017)	√	√		√		
The current study	√	√	√			

decision variable is x_{ijv} that is related to going or not going a specific plane v from origin i to destination j. According to the objective of this study, simultaneously considering three relevant options including booking, cancelation, and overloading, we have denoted these options by b, c and o, respectively. Each considered option includes a penalty for the passenger.

The presented model in this study consists of the objective function (introduced by z) which calculates the amount of revenue that an airline can receive from the flights between cities with various kinds of airplanes. P_{jvb} refers to the profit from buying a ticket in the situation of booking, P_{jvc} represents the penalty of canceling a ticket by a passenger and P_{jvo} denotes the penalty of overloading more than the allowed amount per kilogram by a passenger. These parameters of the model are probabilistic where the ticket cancelation, booking, and overloading limits are being considered and we show their probabilities by $\Pr(o)$, $\Pr(c)$ and $\Pr(b)$. Eventually, $N_{x_{ij}}$ introduces the number of passengers in each plane, which is a parameter, and $P_{x_{ij}}$ refers to the price of a ticket for a specific flight from origin i to destination j. The proposed mathematical optimization model is presented below:

$$\text{Max } z = \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{c=1}^{C}\sum_{b=1}^{B}\sum_{o=1}^{O}\sum_{v=1}^{V} x_{ijv}N_{x_{ij}}(P_{jvc} + P_{jvb} + P_{jvo}) \tag{1}$$

such that

$$x_{ijv} \times \Pr(c) \times P_{jvc} \times N_{x_{ij}} \leq G_{ijvc} \quad \forall v \in V, c \in C, i \in I, j \in J \tag{2}$$

$$x_{ijv} \times \Pr(b) \times P_{jvb} \times N_{x_{ij}} \leq D_{ijvb} \quad \forall b \in B, v \in V, i \in I, j \in J \tag{3}$$

$$x_{ijv} \times \Pr(o) \times P_{jvo} \times N_{x_{ij}} \leq H_{ijvo} \quad \forall v \in V, o \in O, i \in I, j \in J \tag{4}$$

$$x_{ijv} \in \{0,1\} \quad \forall v \in V, i \in I, j \in J \tag{5}$$

Equations 1 to 5 demonstrate the mathematical formulation for revenue optimization. Equation 1, which represents the objective function, seeks the maximum revenue of the airline considering booking and penalties of overloading and cancelation options. The first constraint, Eq. 2, is associated with the penalty of cancelation as well as its probability while Eq. 3 refers to the probability of booking the seats of a plane. The probability of overloading is incorporated in Eq. 4. Finally, Eq. 5 indicates that the decision variables are binary.

4 A case study of the Iranian Airline Industry

4.1 Problem description

Company X is the oldest and best-known airline in Iran. This company transfers passengers to most cities in Iran as well as cities beyond its borders. However, after some years as a monopolist, the case company commenced facing challenges from other airlines. Hence, in such a competitive market, RM is vital for the company's survival. Table 3 shows the company's plane types and their capacities.

This airline has many routes for transferring passengers between different locations. However, most of these routes use the same planes. In this study, we focused our planning

Table 2 Methods of solving revenue management problems

Author(s)	Considered options													
	Dynamic programming	Decomposition	Channel control	Utility function	Robustness	Isoelastic multiplicative price-demand function	Hellerman capacity option	Black–scholes	Decomposition	Nash equilibrium	Nested network	Bid price	Queue system	Markov decision Process
Aydin and Birbil (2018)	√	√												
Tian et al. (2018)			√											
Gönsch (2017)				√	√									
Kyparisis and Koulamas (2018)						√								
Lin et al. (2017)							√	√						
Li and Pang (2017)									√					
Grauberger and Kimms (2016b)										√				
Pimentel et al. (2018)											√	√		
Wei et al. (2013)													√	
Sierag et al.(2015)	√													√

Table 3 Types of planes

Type of plane	Number of passengers
McDonnell Douglas MD-82	140
Fokker 100	104
Airbus A320-200	144
Airbus A310-300	198
Airbus A300-600R	239
Airbus A300B4-200	236

on several megacities with a high travel demand: Tabriz, Shiraz, Mashhad, Isfahan and Tehran as a hub airport for each exiting origin–destination (OD) pairs, in the sense that all the considered flights in this research departure from Tehran and after arriving at the mentioned destination travel back to Tehran. Table 4 depicts the prices of tickets from each evaluated city as follows:

Following the objective of the study, the proposed mathematical model related to the revenue optimization model of the considered company is explained in detail in the next section.

4.2 Assumptions of the model

In the real world, there are too many restrictions to fully solve the problem. To be able to solve this problem, these restrictions must be taken into account. Hence, the assumptions used in our model are as follows:

- All of the passengers are adults: Since the price of children's tickets is less than that for adults, calculations and estimations as to how many children travel by plane and finding the function distribution of such a situation are very difficult. Therefore, we assumed all of the passengers to be adults.
- We assumed the price to be stable, even though on some days, the price of tickets may increase dramatically. Although the revenue of airlines increases significantly on such days, such an occurrence is based on chance.
- If the number of passengers is below 80% of the plane's capacity, then that flight will be canceled. Airlines have computed that the flight is not worth it if under 80% of the seats are occupied, because the income that would be earned is not enough to cover maintenance, fuel, and other concerns.
- The average overloading weight is eight kilograms: Since the weight of some passengers' freight is within the normal range and that of others is higher, the calculation of how many passengers are over the limit and by how much is very difficult. Therefore, we considered eight kilograms to be the average weight for overloading based on the guidelines of Company X.
- Since in summer the demand for air travel in Iran increases dramatically owing to school holidays (the whole summer) and vacation travel, we consider only this season (90 days of operations) for our calculations.
- Only one airline of the many airlines in Iran is considered in this study (Company X).
- Flights are two-way and the planes return to parking for maintenance checks after returning to the original location. On some plane trips, the journey is not as short. Some planes go from their original location to the first destination, then to the next destination and then return to their original location. Since the other destinations will not be megacities, we assumed that the plane would travel to only one location and back again.

Table 4 Prices of flights

City	Price ($)
Isfahan	76
Shiraz	85
Tabriz	64
Mashhad	69

4.3 Mathematical model

4.3.1 Objective function

First, we seek to maximize revenue. We decide whether a certain type of plane should be allocated to a flight and develop a variable that indicates the allocation of a plane.

4.3.2 Booking option for seats

This airline has six types of airplanes with different capacities. Moreover, the price of tickets is different based on the demand for and on the length of the flights. Hence, the revenue from buying, booking, and canceling seats can be described as follows. In this section, the planes followed a simple rule: If more than 80% of seats are occupied, then they will take off, and if less than 80% of seats are occupied, then the flight will be canceled. Therefore, the rate of occupation was from 80 to 100%, and the number of seats on each plane multiplied this rate. Moreover, the ticket to each city has a unique price.

$$x_{ijv} \times \Pr(b) \times P_{jvb} \times N_{x_{ij}} \leq D_{ijvb} \quad \forall b, v, i, j$$

4.3.3 Penalty for cancelation

There are many reasons why some passengers cancel their flights, and most airlines have the same rules regarding this issue. The penalties for cancelation depend on the time of cancelation: If it is close to the departure time, then the rate increases dramatically. In this study, we assume that this penalty rate is the average of the lowest and highest penalty rates, so between 0 and 20%.

$$x_{ijv} \times \Pr(c) \times P_{jvc} \times N_{x_{ij}} \leq G_{ijvc} \quad \forall v, c, i, j$$

4.3.4 Overloading

Another kind of resource from which airlines profit is overloading. Most passengers usually buy many souvenirs during their travels and, therefore, the luggage weight on the return flight may be higher than the normal weight. In this study, we assume that the average weight of overloading is eight kilograms. Then, this average was multiplied by the number of passengers, the probability of overloading, and the penalty cost for 2$ per kilogram. The following formula shows this computation.

$$x_{ijv} \times \Pr(o) \times P_{jvo} \times N_{x_{ij}} \leq H_{ijvo} \quad \forall v, o, i, j$$

4.3.5 Total expected profit of cancelation, booking, and overloading

This section presents a model of the profits gained from the options we have considered in this study. The total expected profit of cancelation deals with the number of seats that are permitted to be canceled and the related penalties. The total expected profit of booking means that all seats of a plane are occupied. The total expected profit of overloading points out that most of the passengers have an overload and therefore have to pay a penalty but the weights from overloading do not lead to a violation of the weight threshold.

$$\text{Max } z : \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{c=1}^{C} \sum_{b=1}^{B} \sum_{o=1}^{O} \sum_{v=1}^{V} x_{ijv} N_{x_{ij}} (P_{jvc} + P_{jvb} + P_{jvo})$$

It is remarkable that Company X has 54 airplanes, which have more than 300 flights daily. Furthermore, the number of seats in each airplane varies from 104 to 239 seats. Considering the large size of the considered problem and the stochastic parameters of the presented model, we deal with an integer-programming model. Due to the computational complexity, it is not feasible to determine exact solutions to solve the problem. Therefore, we resorted to BDE as a viable metaheuristic approach (Table 5).

5 Solution methodology

5.1 The constraint handling approach

Since the model used in this paper has several constraints, we face a constrained optimization problem. Generally, a constrained optimization problem has two main components, an objective function and "n" constraints. Let the general structure of the problem be as Eq. 6:

$$\begin{aligned} &\text{Minimize} \quad f(x), \quad x = (x_1, x_2, \ldots, x_m) \\ &\text{Subject to:} \quad g_i(x) \leq 0 \quad i = 1, 2, \ldots, n \end{aligned} \tag{6}$$

where $f(x)$ is the objective function of the problem and $g_i(x)$ represent the constraints of the optimization problem (e.g., Eqs. (2)–(4) of the problem formulation on p. 7), n refers to the number of constraints and m is the number of variables. In order to change this problem to an unconstrained optimization problem, the constraints of the problem can be added to the objective function using a penalty approach. While the penalty function imposes punishment to the constraints violations, hence, its basic form to be minimized can be defined as follows:

$$\text{Min} \hat{f}(x) = f(x) + \lambda \cdot V(x) \tag{7}$$

$$V(x) = \sum_{i=1}^{n} \max(0, g_i(x))^2 \tag{8}$$

$\hat{f}(x)$ represents the objective function modified by a penalty function, $f(x)$ is the original objective function, λ is a constant and denotes the penalty parameter, $g_i(x)$ is the ith constraint of the model and $V(x)$ computes the violation from it.

5.2 Solution procedure of BDE algorithm

Like many exiting metaheuristic algorithms, the family of evolutionary algorithms has become one of the most useful and efficient optimization algorithms inspired by the concepts found in nature. The BDE algorithm is one of the well-known branches of evolutionary algorithms and has many benefits such as the simplicity of usage, accuracy, and reasonable solution time (Doerr and Zheng 2020). In this section, we elaborate on the steps of the applied BDE algorithm for solving the proposed model. For an L-dimensional optimization problem, the algorithm starts with producing an initial population $p(0)$ comprising N individuals which are generated randomly. The algorithm has three steps including mutation, crossover, and selection.

First of all, we define the basic parameters and inputs of the algorithm like objective function, constraints, number of variables, population size (nPoP), the maximum number of iterations and crossover rate. Thereafter, in the mutation

Table 5 Formulas for calculating each option, type, and the city of flights

Type of plane	Booking	Cancelation	Overloading	City
McDonnell Douglas MD-82	$P_{\mathrm{IbMD}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{IbMD}}\mathrm{d}r_i$	$P_{\mathrm{IcMD}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{IcMD}}\mathrm{d}r_i$	$P_{\mathrm{IoMD}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{IoMD}}\mathrm{d}r_i$	Isfahan
Fokker 100	$P_{\mathrm{Ibf100}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{Ibf100}}\mathrm{d}r_i$	$P_{\mathrm{Icf100}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{Icf100}}\mathrm{d}r_i$	$P_{\mathrm{Iof100}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{Iof100}}\mathrm{d}r_i$	
Airbus A320-200	$P_{\mathrm{IbA200}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{IbA200}}\mathrm{d}r_i$	$P_{IcA200}(r_i \geq 0) = \int_{0}^{0.2} P_{IcA200}dr_i$	$P_{\mathrm{IoA200}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{IoA200}}\mathrm{d}r_i$	
Airbus A310-300	$P_{\mathrm{IbA300}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{IbA300}}\mathrm{d}r_i$	$P_{\mathrm{IcA300}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{IcA300}}\mathrm{d}r_i$	$P_{\mathrm{IoA300}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{IoA300}}\mathrm{d}r_i$	
Airbus A300-600R	$P_{\mathrm{IbA600}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{IbA600}}\mathrm{d}r_i$	$P_{\mathrm{IcA600}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{IcA600}}\mathrm{d}r_i$	$P_{\mathrm{IoA600}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{IoA600}}\mathrm{d}r_i$	
Airbus A300B4-200	$P_{\mathrm{IbB200}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{IbB200}}\mathrm{d}r_i$	$P_{\mathrm{IcB200}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{IcB200}}\mathrm{d}r_i$	$P_{\mathrm{IoB200}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{IoB200}}\mathrm{d}r_i$	
McDonnell Douglas MD-82	$P_{\mathrm{TbMD}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{TbMD}}\mathrm{d}r_i$	$P_{\mathrm{TcMD}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{TcMD}}\mathrm{d}r_i$	$P_{\mathrm{ToMD}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{ToMD}}\mathrm{d}r_i$	Tabriz
Fokker 100	$P_{\mathrm{Tbf100}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{Tbf100}}\mathrm{d}r_i$	$P_{\mathrm{Tcf100}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{Tcf100}}\mathrm{d}r_i$	$P_{\mathrm{Tof100}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{Tof100}}\mathrm{d}r_i$	
Airbus A320-200	$P_{\mathrm{TbA200}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{TbA200}}\mathrm{d}r_i$	$P_{\mathrm{TcA200}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{TcA200}}\mathrm{d}r_i$	$P_{\mathrm{ToA200}}(r_i \geq 0) = \int_{0.8}^{1} P_{\mathrm{ToA200}}\mathrm{d}r_i$	
Airbus A310-300	$P_{\mathrm{TbA300}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{TbA300}}\mathrm{d}r_i$	$P_{\mathrm{TcA300}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{TcA300}}\mathrm{d}r_i$	$P_{\mathrm{ToA300}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{ToA300}}\mathrm{d}r_i$	
Airbus A300-600R	$P_{\mathrm{TbA600}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{TbA600}}\mathrm{d}r_i$	$P_{\mathrm{TcA600}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{TcA600}}\mathrm{d}r_i$	$P_{\mathrm{ToA600}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{ToA600}}\mathrm{d}r_i$	
Airbus A300B4-200	$P_{\mathrm{TbB200}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{TbB200}}\mathrm{d}r_i$	$P_{\mathrm{TcB200}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{TcB200}}\mathrm{d}r_i$	$P_{\mathrm{ToB200}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{ToB200}}$	
McDonnell Douglas MD-82	$P_{\mathrm{MbMD}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{MbMD}}\mathrm{d}r_i$	$P_{\mathrm{McMD}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{McMD}}\mathrm{d}r_i$	$P_{\mathrm{MoMD}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{MoMD}}\mathrm{d}r_i$	Mashhad
Fokker 100	$P_{\mathrm{Mbf100}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{Mbf100}}\mathrm{d}r_i$	$P_{\mathrm{Mcf100}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{Mcf100}}\mathrm{d}r_i$	$P_{\mathrm{Mof100}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{Mof100}}\mathrm{d}r_i$	
Airbus A320-200	$P_{\mathrm{MbA200}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{MbA200}}\mathrm{d}r_i$	$P_{\mathrm{McA200}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{McA200}}\mathrm{d}r_i$	$P_{\mathrm{MoA200}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{MoA200}}\mathrm{d}r_i$	
Airbus A310-300	$P_{\mathrm{MbA300}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{MbA300}}\mathrm{d}r_i$	$P_{\mathrm{McA300}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{McA300}}\mathrm{d}r_i$	$P_{\mathrm{MoA300}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{MoA300}}\mathrm{d}r_i$	
Airbus A300-600R	$P_{\mathrm{MbA600}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{MbA600}}\mathrm{d}r_i$	$P_{\mathrm{McA600}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{McA600}}\mathrm{d}r_i$	$P_{\mathrm{MoA600}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{MoA600}}\mathrm{d}r_i$	
Airbus A300B4-200	$P_{\mathrm{MbB200}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{MbB200}}\mathrm{d}r_i$	$P_{\mathrm{McB200}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{McB200}}\mathrm{d}r_i$	$P_{\mathrm{MoB200}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{MoB200}}\mathrm{d}r_i$	
McDonnell Douglas MD-82	$P_{\mathrm{SHbMD}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{SHbMD}}\mathrm{d}r_i$	$P_{\mathrm{SHcMD}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{SHcMD}}\mathrm{d}r_i$	$P_{\mathrm{SHoMD}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{SHoMD}}\mathrm{d}r_i$	Shiraz
Fokker 100	$P_{\mathrm{SHbf100}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{SHbf100}}\mathrm{d}r_i$	$P_{\mathrm{SHcf100}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{SHcf100}}\mathrm{d}r_i$	$P_{\mathrm{SHof100}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{SHof100}}\mathrm{d}r_i$	
Airbus A320-200	$P_{\mathrm{SHbA200}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{SHbA200}}\mathrm{d}r_i$	$P_{SHcA200}(r_i \geq 0) = \int_{0}^{0.2} P_{SHcA200}dr_i$	$P_{\mathrm{SHoA200}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{SHoA200}}\mathrm{d}r_i$	
Airbus A310-300	$P_{\mathrm{SHbA300}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{SHbA300}}\mathrm{d}r_i$	$P_{\mathrm{SHcA300}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{SHcA300}}\mathrm{d}r_i$	$P_{\mathrm{SHoA300}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{SHoA300}}\mathrm{d}r_i$	
Airbus A300-600R	$P_{\mathrm{SHbA600}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{SHbA600}}\mathrm{d}r_i$	$P_{\mathrm{SHcA600}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{SHcA600}}\mathrm{d}r_i$	$P_{\mathrm{SHoA600}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{SHoA600}}\mathrm{d}r_i$	

Table 5 continued

Type of plane	Booking	Cancelation	Overloading	City
Airbus A300B4-200	$P_{\mathrm{SHbB200}}(r_i \geq 0.8) = \int_{0.8}^{1} P_{\mathrm{SHbB200}} \mathrm{d}r_i$	$P_{\mathrm{SHcB200}}(r_i \geq 0) = \int_{0}^{0.2} P_{\mathrm{SHcB200}} \mathrm{d}r_i$	$P_{\mathrm{SHoB200}}(r_i \geq 0) = \int_{0}^{0.03} P_{\mathrm{SHoB200}} \mathrm{d}r_i$	

phase, for each individual solution (parent), X_j^g, a mutant Y_j^g is generated. Then, the base vector $X_{r_1}^g$ is produced by using the generated indices (r_1, r_2, r_3). In the second phase, crossover, the trial vector U_j^g is generated via a crossover between the mutant and a parent. The last phase is the selection of offspring-parent X_j^{g+1}. If the trial vector is as good as the parent vector concerning fitness, then it substitutes for the parent. Otherwise, the parent will join the next generation. In this step, the best solution will be updated until the algorithm reaches an optimal solution. The procedure of the BDE algorithm is given in Table 6.

After running this model with MATLAB version 2015(b) with a PC CPU core (i) 3 and 5 GB RAM, the results were computed. The total value of revenue earned from the flights was 2.4 million dollars. The computation time was 29 s. nPop (the population size) in this study was 1000. The maximum number of iterations was set to 100 as no significant further improvements were observed with more iterations. The crossover rate of the problem was set to 0.3. Out of approximately 24 flights between these cities, our results showed that only seven planes would not fly in order to obtain the best revenue. This means that 17 planes can transfer passengers between these cities. Table 7 demonstrates the results of our analysis. In Fig. 1, the results of the BDE algorithm for different iterations and the obtained best cost values are reported. As we see in Fig. 1, the obtained solutions (best costs) from the first to the 100 iterations demonstrate that our employed algorithm has reached some improvements in terms of the quality of solutions. Additionally, the blue line indicates a decreasing trend which reveals the fact that the best cost values are diminishing and the algorithm has performed well.

6 Analysis of the results

The obtained findings of the study using the BDE algorithm indicate that among the four megacities, Shiraz should suspend three return flights from Tehran to Shiraz. McDonnell Douglas MD-82, Fokker 100, and Airbus A320-200 airplanes should not be used in these cities. The main reason for this is that these planes have low capacities compared to other types of planes. The number of passengers that want to travel by plane from Tehran to Shiraz and back is not very large, so the types of planes noted above and big planes with high capacities are sufficient for transferring passengers on these routes. The next highest number of flight cancelations was in Tabriz with two flight cancelations. The two kinds of planes to be allocated for this city are the Airbus A300B4-200 and the Fokker 100. Therefore, the case company should not use the planes that have the lowest or the highest capacity. This result indicates that Tabriz always has the same number of passengers and the distribution function of these passengers is uniform.

The cities Mashhad and Isfahan had the same number of plane cancelations on one flight, but the types of planes are different. In Mashhad, the McDonnell Douglas MD-82 should be canceled and the Airbus A300-600R should not fly to Isfahan in order to achieve the best revenue. These differences indicate that the demand for flights from Tehran to Mashhad and back is very high and that planes with high capacities need to be allocated instead of planes with lower capacities. However, the result was opposite for Isfahan, where people have less need to use planes for many reasons, such as the availability of buses, which serve as suitable methods of transport at a lower price than travel by airplane. Hence, the big planes should be allocated to the other cities in order to increase the revenue of the airline company.

The analysis of the types of planes showed that the McDonnell Douglas MD-82 is the type of plane that brings in the lowest revenue to Company X and is also the most frequently canceled type of plane, with a cancelation frequency of four times for planes with low capacities versus two times for larger planes during 90 days of operations. The results of this study revealed that there is a very high demand for airline's planes in megacities and if this airline does not focus on this matter, it will lose customers and its revenue will decrease.

7 Comparative analysis

In this section, in order to check the efficiency of our applied solution approach, BDE algorithm, we solved our RM problem with two different well-known metaheuristic optimization techniques including the harmony search (HS) algorithm (Yang 2009) and cultural algorithms (CA)

Table 6 Procedure of BDE algorithm

Line	Explanation
	Set parameters: Mu = Mutation probability, Cr = Crossover probability, population size (nPop) = 1000, crossover rate = 0.3, number of iterations (g) = 100.
1	Randomly generate primal population $p(0) = \{X_j^0,\ j = 1,\ldots,1000\}$
2	**for** each g = 1,2…,100 **do**
3	**for** j = 1,2…,1000 **do**
	% Mutation
4	Create three mutually distinct indices r_1, r_2, r_3 from {1,2,…,1000}\{j}
5	Produce a random number for each $k \in \{1,\ldots,F\}$
	compute the best cost of the objective function and delineate the mutant Y_j^g and through
6	For $k \in \{1,\ldots,F\}, Y_{j,k}^g = \begin{cases} 1 - Y_{r1,k}^g, & \text{if } Y_{r2,k}^g \neq Y_{r3,k}^g, \text{and the random number} < \text{Mu}; \\ Y_{r1,k}^g & \text{Otherwise} \end{cases}$
	%Binomial Crossover
7	Generate a random number for each k∈{1,...,F}
	Update the best cost and determine the trial vector U_j^g using
8	For $k \in \{1,\ldots,F\}, U_{j,k}^g = \begin{cases} Y_{j,k}^g & \text{if the random number} \leq \text{Cr}; \\ X_{j,k}^g & \text{Otherwise} \end{cases}$
	%Selection
	Update the best cost and choose X_j^{g+1} through
10	$X_j^{g+1} = \begin{cases} U_j^g & \text{if } f\left(U_j^g\right) \geq f\left(X_j^g\right) \\ X_j^g & \text{Otherwise} \end{cases}$
12	**end for**
13	**end for**

(Reynolds 1994), which both are related to the evolutionary computation context and more compatible with the stochastic nature of this study's data. We compared the obtained results of our proposed model solved by the BDE algorithm, HS and CA in terms of the time of computation, the quality of solution for the objective function as well the robustness and accuracy of the results. It is noteworthy that we solved our revenue optimization model by HS and CA exactly with the same properties applied in the solving procedure of the BDE algorithm. Figures 2 and 3 depict the trend of yielding the best costs in both algorithms for 100 iterations.

From a comparison point of view, Table 8 demonstrates that the time of reaching good solutions in HS, CA and BDE are 117.5, 120 and 29 s, respectively, and in terms of the solution time, BDE remarkably outperformed than the other two algorithms. The next comparison criterion was the total value of revenue using different solution techniques that results in 2.3, 1.14 and 2.4 for HS, CA and BDE, respectively. Although the cultural algorithm gained smaller revenue values, the respective amount for HS and BDE was close to each other. The other considered ground was according to the acceptance or rejection of flights according to the type of plane, which is tied to the objective of this study. The analysis of this criterion also reveals that the accuracy of solutions by the BDE algorithm is higher than for the other two methods.

On one hand, although the revenue value of HS and BDE are similar to each other, the HS algorithm suggests that all the flights with diverse kinds of planes are accepted for the case company while we have a more accurate and separable result employing the BDE algorithm. On the other hand, the cultural algorithm recommended that the flights with some specific planes should be canceled, nonetheless, since most of the advised planes were of the planes that have higher capacity, thus, canceling flights with the determined planes could result in diminishing the total revenue of the airline company. Consequently, the solutions given by the cultural algorithm were not viable and applicable. Overall, comparing the solutions under all three criteria explicitly demonstrated that the BDE

Table 7 Final obtained results

Type of plane	Accept/reject	City
McDonnell Douglas MD-82	Accept	Isfahan
Fokker 100	Accept	
Airbus A320-200	Accept	
Airbus A310-300	Accept	
Airbus A300-600R	Reject	
Airbus A300B4-200	Accept	
McDonnell Douglas MD-82	Reject	Tabriz
Fokker 100	Accept	
Airbus A320-200	Accept	
Airbus A310-300	Accept	
Airbus A300-600R	Accept	
Airbus A300B4-200	Reject	
McDonnell Douglas MD-82	Reject	Mashhad
Fokker 100	Accept	
Airbus A320-200	Accept	
Airbus A310-300	Accept	
Airbus A300-600R	Accept	
Airbus A300B4-200	Accept	
McDonnell Douglas MD-82	Reject	Shiraz
Fokker 100	Reject	
Airbus A320-200	Reject	
Airbus A310-300	Accept	
Airbus A300-600R	Accept	
Airbus A300B4-200	Accept	

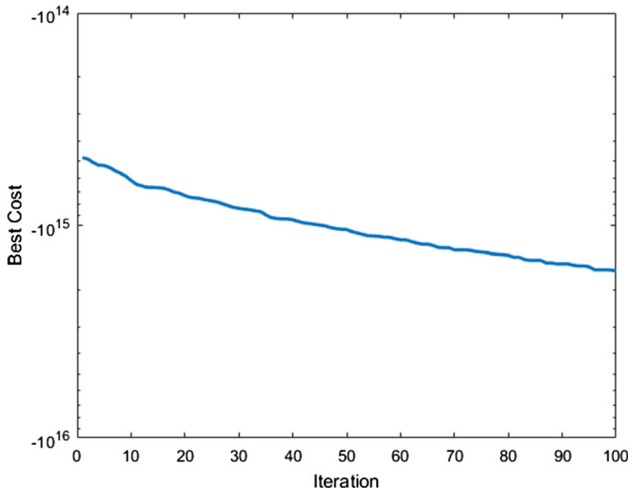

Fig. 1 Solutions obtained by the BDE algorithm

algorithm produces higher quality, more accurate and applicable solutions.

For getting further reliability of the suggested BDE algorithm in comparison with HS and CA, we performed multiple runs (30 repetitions) of the algorithms with the given problem instance to consider stochastic effects influencing the performance of these algorithms. These results were evaluated using a *t* test regarding statistical significance.

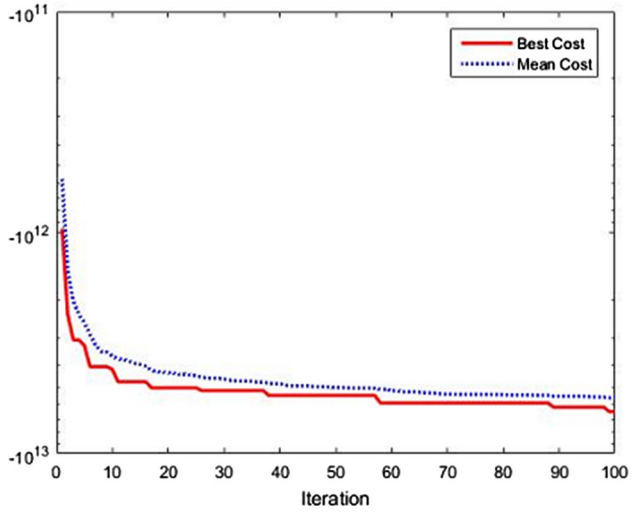

Fig. 2 Solutions obtained by the HS algorithm

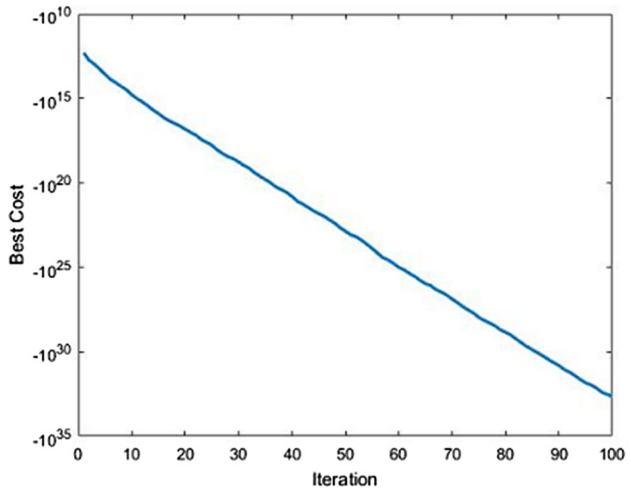

Fig. 3 Solutions obtained by the cultural algorithm

The *t* test checks for a hypothesized mean difference between the considered methods' results of 0. We apply a two-sample *t* test assuming unequal variances. The probability of the above null hypothesis (*p* values) is 7.60749E-05 for HS and 7.4543E-05 for CA. Therefore, the null hypothesis must be rejected for both comparison methods, HS and CA, under all reasonable settings of alpha (error probability of rejecting the null hypothesis), e.g., for alpha = 0.05. The observed differences in performance between the methods are, therefore, statistically significant.

Table 8 Comparison of the derived results using different solution algorithms

Algorithms	Time (Second)	Revenue value (million $)	Acceptance of flights based on the type of plane
HS	117.5	2.3	All the planes
CA	120	1.14	All the planes except Fokker 100 and Airbus A300B4-200 planes for Isfahan, Airbus A310-300 and Airbus A320-200 for Shiraz and Airbus A320-200 for Mashhad
BDE	29	2.4	According to Table 6

8 Concluding remarks

RM is an essential and effective tool that allows managers to make the appropriate decisions in order to maximize revenue. Following this objective, managers have to consider all aspects of the problem, where some of the data are deterministic while other data are stochastic. For competitive markets in the airline business, the tool developed in this study could help airlines to compete better in the market.

In this study, we aimed to help the managers of Company X to optimize the profit of the airline. We proposed a mathematical model that takes into account the overload of passengers' luggage and cancelation. The DE algorithm was used to solve this problem. Company X was analyzed to show how an airline could allocate its planes to megacities in order to achieve superior revenue management. In addition, various options that help to increase revenue were also considered in this study. These options were penalties due to ticket cancelation and overloading. Using the stochastic data, we considered the probabilities of booking, overloading and cancelation in our revenue model to make it more realistic as uncertainty is an integral part of modeling such dynamic systems. The scale of the problem was large, and this model was run for 1000 flights for each of the four cities with a different type of plane. This model cannot be developed into a closed-form solution. Because the problem is NP-hard, an evolutionary algorithm such as DE is well suited to mitigate the drawbacks of previous methods and derive good solutions with rather small computation time. After running the model and obtaining our results, we provided some suggestions and insights for the management of the company under investigation. Adding more options as well as novel options to future RM models may be an interesting idea for further studies.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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